MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 105 No. 5

MEETING OF 1945 OCTOBER 12

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Francis Holden, 1 Waterloo Terrace, Ashton, Preston, Lancashire; and
Rev. Richard Calvert Wood, M.A., 55 Castle Street, Hertford.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Geoffrey Vernon Ball, 20 Marshall Grove, Erdington, Birmingham, 23 (proposed by J. J. Shaw);

Robert Stuart Devonald, B.Sc., 11 Westville Road, Cardiff, S. Wales (proposed by J. Rees);

John Harold Ellison, M.A., A.N.I.A., Woodspeen Grange, near Newbury, Berkshire (proposed by H. R. Hulme);

Jesse L. Greenstein, Assistant Professor of Astrophysics in the University of Chicago, Yerkes Observatory, Williams Bay, Wis., U.S.A. (proposed by S. Chandrasekhar);

William Albert Hiltner, Yerkes Observatory, Williams Bay, Wis., U.S.A. (proposed by S. Chandrasekhar);

Kkateeb Hyder Hussain, B.Sc., H.E.H. the Nizam's Electricity Department, Hyderabad, Deccan, India (proposed by Amin Jung);

Percy John Edward Maycock, 52 Donald Road, Uplands, Bristol, 3 (proposed by F. J. Hargreaves);

George Thomas Smith-Clarke, Shenandoah, Stoneleigh Road, Gibbet Hill, Coventry, Warwickshire (proposed by J. D. F. Williams); and

Thomas Alexander Thomson, B.Sc., 4 Cromwell Street, Wakari, Dunedin, New Zealand (proposed by D. C. Berry).

Seventy-six presents were announced as having been received since the last ordinary meeting, including:—

German Astronomical Computer, ARG I (presented by Wing-Commander R. S. J. Edwards);

The astronomical books of the late Mr. A. R. Hinks (presented by his sons);

E. Strömgren, *Ole Rømer som Astronom* (presented by the author); and

M. Waldmeier, *Sonne und Erde* (presented by the author).

Professor Bengt Edlén delivered the George Darwin Lecture (see No. 6), taking as his subject "The Origin of the Emission Lines in the Coronal Spectrum".

At a Special General Meeting of Fellows, held immediately following the Ordinary Meeting, the following alteration of Bye-law 23 and the repeal of Bye-laws 41A and 87 and a motion with regard to the dates and times of meetings were proposed severally by Mr. D. H. Sadler, Secretary, on behalf of the Council, and seconded by Dr. R. Stoneley in each case:

> 23. Any Fellow may, at his entrance, compound for his contributions by the payment of thirty guineas, exclusive of his admission fee; or he may at any time afterwards in the ensuing five years (all sums then due being first paid) compound for his *subsequent* annual contributions by a like payment of thirty guineas. After the first five years the Composition Fee will be reduced by one guinea for each year of Fellowship in excess of five years, to a minimum fee of five guineas, under the same conditions.

shall be amended to read as follows:

> 23. Any Fellow may, at his entrance, compound for his annual contributions, exclusive of his admission fee, by the payment of a *Composition Fee* of forty-five guineas less one half-guinea for each year of age (on the first day of January in the year of entrance) in excess of thirty; or he may at any time afterwards (all sums then due being paid) compound for his *subsequent* annual contributions, in which case the *Composition Fee* then payable will be the *Composition Fee* as calculated for the date of entrance, reduced by one guinea for each year of Fellowship to a minimum of five guineas.

The Secretary gave a short explanation of the reasons for the alteration.

A Fellow raised the point that, if the Bye-law were passed, it would place those Fellows who had already compounded in an anomalous position, as they would have paid Composition Fees which were not in accord with the present Bye-laws. The Secretary promised that Council would consider this matter and would, if necessary, move an explanatory addition to the Bye-law at the Annual General Meeting of 1946 February 8. With this promise, the Bye-law, as amended, was carried unanimously.

> 41A. For the duration of the present lighting restrictions, and during the present emergency, business at ordinary meetings of the Society shall commence at such earlier hour as Council may direct.

> 87. Notwithstanding the present Bye-laws, the Council may, at any time during the present emergency when the normal procedure of calling a Special General Meeting is impracticable, suspend or abrogate such Bye-laws as may appear necessary or desirable in the interests of the Society.

These were repealed without opposition.

> "That notwithstanding the Bye-laws of the Society, no Ordinary Meeting shall be held in January 1946 and the meetings in November and December 1945 and February 1946 shall be held, as already announced, at half-past four o'clock."

The Secretary explained the circumstances and reasons for the change and the motion was carried unanimously.

MEETING OF 1945 NOVEMBER 9

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

John Geoffrey Drewe Pratt, 4 Branstone Road, Kew Gardens, Richmond, Surrey; and

Frank Melson Wingate, Malcombe, Pine Wood Road, Bournemouth, Hampshire.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Kenneth Douglas Adams, c/o National Broadcasting Service, Invercargill-Dacre R.D., Southland, New Zealand (proposed by D. C. Berry);

Gerard P. Kuiper, Yerkes Observatory, Williams Bay, Wis., U.S.A. (proposed by J. H. Oort);

Bert Walter McMullen, 3 Fiveways, Ickenham Road, Ruislip, Middlesex (proposed by W. E. Candler);

Limmye Vernon Robinson, Department of Mathematics, Emory University, Georgia, U.S.A. (proposed by H. H. Plaskett); and

George Philip Sidney Streatfeild, B.A., 1 Alverton Place, Penzance, Cornwall (proposed by J. E. Littlewood).

Forty-one presents were announced as having been received since the last ordinary meeting, including:—

H. P. Coster, *The Gravity Field of the Western and Central Mediterranean* (presented by the author);

M. Minnaert, *Light and Colour in the Open Air* [translation] (presented by the author); and

Total Eclipse of the Sun 1945 July 9, four photographs taken and presented by the Royal Canadian Air Force.

ON THE CONSTITUTION OF TRUMPLER'S STAR NGC 6871,5

Jaakko Tuominen *

(Communicated by the Secretaries)

(Received 1945 May 14)

Summary

In order to explain the relatively low observed luminosities of Trumpler's stars, as compared with the luminosities which follow from the theory of purely radiative transfer of heat throughout the star (Eddington's mass-luminosity formula), the constitution of NGC 6871,5 has been studied under the assumption that the transfer of heat in the star is partly convectional (due to turbulence), partly radiative. In order to avoid the necessity of very high turbulent speeds, the density must rise relatively fast from the surface inwards. In the innermost part of the star the density is put equal to constant, although it actually may decrease there inwards. The latter phenomenon can follow from the fact that radial convection currents are hampered by a high turbulent viscosity. In the constitution of the star, the heat converted into kinetic energy of the large-scale motions, and the heat produced when turbulence is dissipated by friction, seem to play an important rôle. The turbulence is maintained by large-scale currents and the whole mechanism is kept going by the subatomic heat.

---

1. *Introduction.*—In 1939 the writer started a series of papers on Trumpler's stars. In the first of these papers, published in the *Monthly Notices* †, the theoretical luminosities of Trumpler's stars, as derived from the Eddington mass-luminosity formula, were compared with their observed luminosities. In three subsequent papers, published elsewhere †, a theory was developed, capable of explaining the differences, found in (I), between the theoretical and observed luminosities. In papers (II)–(IV) the star was considered to be in rotation.‡ The rotation necessarily causes large-scale convection currents in the star. In consequence of the great size of all stars, the flow is then turbulent. In a vertically stable outer part, the turbulence then transports heat inwards and reduces the luminosity. That the turbulent transfer of heat in the inner part, because of the vertical instability of the distribution of the density there, generally is directed outwards, and increases the total flux of heat, is no obstacle to the theory. The truth is that in the outer parts of the star a part of the heat is converted into kinetic energy of the large-scale motions. The Sun, for instance, shows large-scale convection currents which cannot be a remnant from primordial times. The turbulent viscosity in the Sun apparently is so high that barely more than a few million years are needed to smooth out the equatorial acceleration of the Sun.§ Hence in the Sun heat is continually converted into kinetic energy of the large-scale motions. As will be shown in section 7 (Table VI), in Trumpler's stars the latter conversion must take place mostly in the outer part of the star. On the other hand, the large-scale convection currents create turbulence, which then is dissipated by friction. In this process the kinetic

* Holder of a grant from the Foundation of Prof. E. N. Setälä, Helsinki.

† (I) = *M.N.*, 100, 86, 1939; (II) = *Z. Astrophys.*, 22, 90, 1942; (III) = *ibid.*, 22, 165, 1942; (IV) = *Ark. Mat., Astr. Fys.*, 30 A, No. 16, 1944.

‡ The deviation from spherical symmetry was still considered to be negligible.

§ Rosseland, *Astrophysik auf atomtheoretischer Grundlage*, p. 90, Berlin, 1931.

energy of the large-scale motions is first converted into kinetic energy of the turbulence, which, finally, is converted into heat. Table VI shows that in Trumpler's stars the conversion of kinetic energy into heat occurs mainly in the inner part of the star. Besides, heat may be lost or gained by the positive or negative work done by the gas when it, due to turbulence, expands or contracts. The whole mechanism is kept going by the subatomic heat, which penetrates the surface and hence is lost by the star.

The star to which the theory will be applied in the present paper is the star with the greatest difference between observed luminosity and the luminosity calculated on the basis of Eddington's theory, i.e. NGC 6871,5. For this star *

$$\log (M/M_\odot) = 2{\cdot}60; \qquad \log (L/L_\odot) = 5{\cdot}01; \qquad \log (R/R_\odot) = 1{\cdot}22.$$

Hence

$$L/L_\odot = 1{\cdot}02 \,.\, 10^5$$

and, in c.g.s. units,

$$M = 7{\cdot}91 \,.\, 10^{35}; \qquad R = 1{\cdot}15 \,.\, 10^{12}.$$

According to Eddington's theory, its lowest possible theoretical luminosity, found for pure hydrogen, is given by [(I)]

$$\log (L/L_\odot)_{\mu=0{\cdot}5} = 6{\cdot}53.$$

Hence the lowest theoretical luminosity is about 33 times higher than the observed luminosity. In the present paper it will be assumed that one-third of the mass of NGC 6871,5 consists of hydrogen and the rest of Russell-mixture. Then the mean molecular weight $\mu = 1$. This should agree with the average conditions in the stars.† We shall thereupon study what kind of composition of the star would make its theoretical mass, luminosity and radius agree with the observed values.

2. *The Fundamental Formulæ of the Problem.*—As in the classical problem of the stellar constitution, the fundamental formulæ which govern the constitution of a nearly spherically symmetrical star are the equation of heat transfer and the equation of hydrostatic equilibrium.‡ When turbulence is present in the star, the equation of heat transfer, as derived in (II) [equations (13) and (14)], can be expressed in the form

$$L_r = -4\pi c \frac{r^2}{\kappa\rho} \frac{d(\frac{1}{3}aT^4)}{dr} + 6\pi \frac{\beta^2}{4-3\beta} r^2 P \,.\, \bar{U}, \tag{1}$$

where [(II), equation (3)]

$$\bar{U} = \tfrac{1}{8} q l \left(\frac{1}{\rho} \frac{d\rho}{dr} - \frac{1}{\gamma P} \frac{dP}{dr} \right) \tag{1'}$$

represents the mean value of the radial component of the turbulent velocity. When relation (1') is inserted in (1), it follows that

$$L_r = -4\pi c \frac{r^2}{\kappa\rho} \frac{d(\frac{1}{3}aT^4)}{dr} + \frac{3}{4}\pi \frac{\beta^2}{4-3\beta} r^2 P \left(\frac{1}{\rho} \frac{d\rho}{dr} - \frac{1}{\gamma P} \frac{dP}{dr} \right) ql. \tag{2}$$

The first member on the right represents the radiative flux of heat and the second member the turbulent flux of heat through a spherical surface of radius r. L_r is the total flux of heat. The quantities c and a represent respectively the velocity of light and the density of radiation at 1° K; ρ, T, P and r are respectively the density, temperature, total pressure and radius vector from the centre; κ is the mass absorption coefficient and γ the ratio of specific heats; q and l respectively represent the mean velocity and the mean free path, i.e. the mixing length, of the turbulence elements. The quantity

* Kuiper, *Ap. J.*, 88, 506 (Table 13), 1938.

† Strömgren, *Z. Astrophys.*, 7, 235, 1933. In fact, the average value $\mu = 1$ is based on the assumption of purely radiative transfer.

‡ It is emphasised that the irregular turbulence does not interfere with the average hydrostatic equilibrium.

$1-\beta$ means the ratio of the radiation pressure to the total pressure. Formula (13) in (II) was written in the form (1) by neglecting the turbulent pressure. In fact, the turbulent pressure seems to be of no great significance in Trumpler's stars [(III), p. 180, and (IV), p. 5]. Hence

$$P=\frac{\mathfrak{R}}{\mu}\rho T+\frac{a}{3}T^4, \tag{3}$$

where $\mathfrak{R}$ is the gas constant and μ the mean molecular weight. $\frac{\mathfrak{R}}{\mu}\rho T$ is the gas pressure and $\frac{1}{3}aT^4$ the radiation pressure. Naturally, formula (1) has been derived under certain other simplifying restrictions too. That, for instance, the transfer of heat by large-scale currents has been neglected, does not seem to lead to any serious error, because the large-scale motions mostly follow level surfaces [(II), p. 93]. For the theory of Trumpler's stars it is important that formula (1) has been derived for an arbitrary $(1-\beta)$-value.*

The equation of hydrostatic equilibrium is, as usual,

$$\frac{dP}{dr}=-G\frac{M_r\rho}{r^2}, \tag{4}$$

where G is the constant of gravitation and M_r, the mass within a sphere of radius r, is given by

$$M_r=4\pi\int_0^r\rho r^2dr. \tag{5}$$

The difference between the classical problem of the constitution of a star in hydrostatic equilibrium with purely radiative transfer of heat, and the present problem, is to be found, first, in the equation of heat transfer [(2)]. The latter equation contains a member due to the transfer of heat by turbulence, which member is rather complicated. Secondly, in the case of purely radiative transfer of heat, the quantity L_r may be expressed as an integral of the generation of subatomic heat per unit mass and time. In cases when turbulence is present, as illustrated in section 1, heat is generated not solely by subatomic processes. On the contrary there are processes which transform heat into other forms of energy. The corresponding expression of L_r will be studied a little further in section 7 [formula (38)].

The mass-luminosity-radius law of Eddington, which is based on a purely radiative transfer of heat, has already been found by Eddington † to be practically independent of the law of generation of heat. Hence, without knowledge of the process of heat generation it has been possible to derive, for instance, the hydrogen content of the stars, from their observed masses, luminosities and radii. In a similar way, it is not necessary to know all the processes in connection with generation and dissipation of heat in the interior of Trumpler's stars; without knowing them interesting properties of Trumpler's stars may be derived.

When the stellar luminosity is calculated from its mass and radius by means of the Eddington mass-luminosity formula, the calculation is actually made under the assumption of the standard model. However, practically the same luminosity will be found if the calculation is based on an even greatly different distribution of the density [(IV), section 2]. The Eddington mass-luminosity formula is based, in fact, almost entirely on the assumption that the heat is transferred by radiation only. Just for this reason, because the Eddington formula did not give the right luminosity for Trumpler's stars (save for one of them), it seems necessary to take into account the turbulent transfer of heat.

* The reason why a factor 1/8 was adopted in formula (1′) will be understood from (II), p. 97, and (IV), p. 17 (footnote).

† *The Internal Constitution of the Stars*, p. 127, Cambridge, 1926.

3. *The Outer Layers of the Star with a Purely Radiative Transfer.*—At the surface, where the density vanishes, the heat transfer must take place by radiation only. This can also be seen from equation (2), in which the second member on the right vanishes, when P, the pressure, vanishes. The transfer of heat then proceeds as a more or less purely radiative process down to some depth below the surface. Hence, in the outer part of the star, the second member on the right of equation (2) may be put equal to zero, and the problem is then the common problem of radiative transfer. Its solution (for the outer layers of the star) has been derived by Chandrasekhar * and by the present writer. The latter has found the following formulæ †, valid in the outer part of the star where M_r and L_r may be considered as independent of r and, besides, the scattering by free electrons may be neglected in comparison to photo-electric absorption:

$$\frac{R}{r}-1=\frac{16\pi acR}{3L}\frac{a}{3\mathfrak{R}}\left(\frac{L}{4\pi cGM}\right)^3\mu\kappa_1{}^2\cdot\tfrac{1}{3}[(\vartheta+1)^3(3\vartheta+1)-1]; \tag{6}$$

$$T=\left[\frac{a}{3\mathfrak{R}}\frac{L}{4\pi cGM}\mu\kappa_1\vartheta(\vartheta+1)\right]^2; \tag{7}$$

$$\rho=\frac{a}{3\mathfrak{R}}\mu\vartheta T^3. \tag{8}$$

Here M, L and R are the total mass, luminosity and radius of the star, expressed in c.g.s. units. κ_1 represents the factor, considered as constant, of $\rho/T^{3\cdot5}$ in the coefficient of absorption,‡

$$\kappa_a=\frac{[25\cdot63](1-X)}{\tau}\frac{\rho(1+X)}{T^{3\cdot5}}. \tag{9}$$

X is the hydrogen content, the helium content being neglected. The factor [25·63] ($=10^{25\cdot63}$) corresponds to the "Case 1" in Morse's paper, i.e. to Russell mixture mixed with an arbitrary amount of hydrogen and helium. The guillotine factor is denoted by τ instead of the longer expression $t/\bar{g}$ used in Morse's paper. Hence

$$\kappa_1=\frac{[25\cdot63](1-X^2)}{\tau}. \tag{10}$$

The quantities μ and X are connected with each other by the formula §

$$X=\frac{(1/\mu)-0\cdot5}{1\cdot5}. \tag{11}$$

Further [formula (8)],

$$\vartheta=\frac{\beta}{1-\beta}\quad\text{or}\quad 1-\beta=\frac{1}{1+\vartheta}. \tag{12}$$

Table I contains the values of log T, log ρ, x and $1-\beta$ for NGC 6871,5, calculated from the above formulæ for different values of ϑ. Column 6 contains log τ as extrapolated or interpolated from Morse's table (Table 3, Case 1). Column 7 gives the photo-electric coefficient of absorption thus calculated, and column 8 the total coefficient of opacity which includes the scattering by free electrons

$$\kappa_s=0\cdot19(1+X). \tag{13}$$

---

* *M.N.*, 96, 647, 1936.

† *Ann. N.Y. Acad. Sci.*, 41, 69–70, or Harvard Reprint, 224, 1941 [formulæ (27), (28) and (22)]. In these formulæ the subscript 0 in ϑ_0 will be dropped.

‡ Morse, *Ap. J.*, 92, 40–41, 1940.

§ Strömgren, *Z. Astrophys.*, 7, 229, 1933.

TABLE I

The Outer Part of NGC 6871,5 with a Purely Radiative Transfer of Heat and $\mu = 1$

1 ϑ	2 $\log T$	3 $\log \rho$	4 x	5 $1-\beta$	6 $\log \tau$	7 $\log \kappa_a$	8 $\log \kappa$	9 $\frac{L_{turb}/L_\odot}{q(l/R)}$
0·0	$-\infty$	$-\infty$	1·000	1·0000	0·00	1·71 *	1·72	0·0
0·1	0·80	0·90–22	1·000	0·9091	·00	1·68	1·69	
0·5	2·47	·60–16	1·000	·6667	·00	1·54	1·55	$-1\ .10^{-11}$
1·0	3·32	·46–13	1·000	·5000	·00	1·42	1·44	$-3\ .10^{-8}$
1·5	3·87	·27–11	1·000	·4000	·00	1·30	1·32	$-1\cdot0.10^{-5}$
2	4·28	·62–10	1·000	·3333	·00	1·22	1·25	$-3\cdot3.10^{-4}$
3	4·88	·60– 8	0·999	·2500	·00	1·10	1·14	$-4\cdot5.10^{-2}$
4	5·32	·06– 6	·998	·2000	·00	1·02	1·06	$-1\cdot4.10^{0}$
5	5·67	·21– 5	·996	·1667	·00	0·94	0·98	$-2\cdot5.10^{1}$
6	5·97	·17– 4	·992	·1428	·15	0·70	0·76	$-3\cdot3.10^{2}$
7	6·22	·99– 4	·986	·1250	·19	0·61	0·68	$-2\cdot3.10^{3}$
8	6·43	·70– 3	·978	·1111	·14	0·64	0·71	$-9\cdot7.10^{3}$
9	6·63	·33– 2	·966	·1000	·13	0·58	0·65	$-5\cdot6.10^{4}$
10	6·80	0·90– 2	0·951	0·0909	0·25	0·43	0·52	$-2\cdot0.10^{5}$

The total coefficient of opacity has been calculated by means of Table 2 in Morse's paper.

Evidently, when the formulæ (6)–(8) were derived, the coefficient of opacity κ was replaced by the quantity $\tau.\kappa_a$. As can be seen from Table I, the logarithm of the latter quantity does not deviate much from $\log \kappa$. Further, because of the low density in that part of the star, M_r and L_r are practically constant. Accordingly we were in every respect entitled to use formulæ (6)–(8) for this part of the star provided only that the transfer of heat by turbulence may be neglected in comparison with the transfer of heat by radiation.

Let us now write the total flux of heat in the outer part of the star without putting the turbulent transfer identically equal to zero. The distribution of the density and temperature were derived so that the radiative flux of heat is equal to the luminosity, when the opacity coefficient is equal to the photo-electric coefficient of absorption multiplied by the guillotine factor. Hence

$$\frac{d(\frac{1}{3}aT^4)}{dr} = -\frac{L}{4\pi c}\frac{\kappa\rho}{r^2},$$

where $\kappa = \tau\kappa_a$. From equation (3) it is easily found that

$$\frac{d\rho}{dr} = \left(\frac{dP}{dr} - \frac{4-3\beta}{4(1-\beta)}\frac{d(\frac{1}{3}aT^4)}{dr}\right)\Big/\frac{\Re}{\mu}T.$$

The above expression of $d(\frac{1}{3}aT^4)/dr$ is now inserted in the latter equation. The expression $d\rho/dr$ thus obtained, and further dP/dr according to formula (4), are then inserted in (2). It is, naturally, suitable to express the mixing length l in units of the stellar radius. Then it follows that

$$L_r = L + \tfrac{3}{4}\pi R\frac{\beta}{4-3\beta}\rho\left[GM\left(\frac{\beta}{\gamma}-1\right)+\frac{L}{16\pi c}\frac{4-3\beta}{1-\beta}.\kappa\right]q(l/R), \qquad (14)$$

* This value is the limiting value for vanishing ϑ.

where, as above, $\kappa = \tau\kappa_a$. The value of γ, a function of $1-\beta$, may be calculated from the formula *

$$\gamma = \beta + \frac{2}{3}\frac{(4-3\beta)^2}{8-7\beta}, \tag{15}$$

applicable for monatomic gas.

The values of the turbulent transfer divided by $q(l/R)$, as found from equation (14), are given in Table I (column 9). The first member in the brackets in (14) is negative and the second one positive. For the ϑ-values from zero to unity both members have nearly the same magnitude. Therefore, because of the rough calculation of the opacity coefficient, the values of the turbulent transfer are obtained only roughly for these ϑ-values.

4. *The Part of the Star with an Inwards Directed Turbulent Transfer.*—To what depth of the stellar interior the turbulent transfer may be neglected in comparison with the radiative transfer cannot be said with certainty. It is clear from formulæ (6)–(8), however, that by purely radiative transfer the density rises so rapidly towards the interior that soon the whole mass of the star will be used up by its outer layers. In order to avoid this, the density must be made to increase less quickly. The radiative transfer then increases—because of the greater transparency of matter per volume—from the observed luminosity value, and, in fact, rather quickly. In order to counterbalance this, a turbulent transfer of heat directed inwards must be superposed on the radiative transfer. Two kinds of difficulties, at least apparent difficulties, are then encountered:

Firstly, to enable the inwards directed turbulent transfer nearly to counterbalance the outwards directed radiative transfer, because of the low density in the outer parts of the star the average turbulent velocity may become very great. Mathematically this state of affairs is easily seen from formula (2), where the second term on the right must nearly counterbalance the first one. In the outer parts of the star P is small and hence for ql big values are needed. This is clearly seen from the calculations in (II) ($-\bar{U}$ increases fast in the vicinity of the surface).

Secondly, if the distribution of the density does not deviate sufficiently from the adiabatic distribution, the average turbulent speed of the matter, or—alternatively—the mixing length, must be correspondingly great. Mathematically also this is clearly seen from formula (2), in which q and l must be big if the expression in parenthesis is not sufficiently great.

Naturally, the first difficulty is overcome by continuing the purely radiative transfer sufficiently deep into the interior, where the density (or pressure) is sufficiently high. In order to remove the other difficulty, we must make the density distribution deviate sufficiently from the adiabatic distribution. The latter deviation must be such that the expression in the parenthesis in equation (2) attains sufficiently high negative values.

In the present paper a certain condition will be chosen for the pressure-density relation, which makes it certain that the expression in parenthesis in equation (2) is negative in all cases. This will be done by putting $1-\beta = 1-\beta' = \text{const.}$ in the spherical shell where the turbulent transfer is directed inwards. Since

$$\frac{\Re}{\mu}\rho T = \beta' P$$

and

$$\tfrac{1}{3}aT^4 = (1-\beta')P, \tag{16}$$

it then follows by elimination of T,

$$\rho = \frac{(a/3)^{1/4}}{\Re/\mu}\frac{\beta'}{(1-\beta')^{1/4}}P^{3/4}. \tag{16'}$$

* Eddington, *I.C.S.*, pp. 190–191.

Insert this in equations (4) and (5) and insert M_r according to (5) into (4). Set further

$$r = Rx \tag{17}$$

and

$$P = \left(\frac{1}{\pi G}\right)^2 \frac{(\Re/\mu)^4}{a/3} \frac{1}{R^4} \frac{1-\beta'}{\beta'^4} u^4 \tag{18}$$

where x and u are new variables and R is the stellar radius. The equation of hydrostatic equilibrium [equation (4)] then assumes the form

$$\frac{du}{dx} = -\frac{1}{x^2}\int_0^x u^3 x^2 dx, \tag{19}$$

which is the known differential equation of the polytropic model of index $n = 3$. This does not mean, however, that our theory would lead to the standard model. The standard model affords a certain $(1-\beta')$-value. Equations (16′) and (18) give

$$\rho = \left(\frac{1}{\pi G}\right)^{3/2} \frac{(\Re/\mu)^2}{(a/3)^{1/2}} \frac{1}{R^3} \frac{(1-\beta')^{1/2}}{\beta'^2} u^3, \tag{20}$$

while (16) and (18) give

$$T = \left(\frac{1}{\pi G}\right)^{1/2} \frac{\Re/\mu}{(a/3)^{1/2}} \frac{1}{R} \frac{(1-\beta')^{1/2}}{\beta'} u. \tag{21}$$

When the region with an inwards directed turbulent transfer starts from a certain depth below the surface, the corresponding $(1-\beta')$-value can be taken from Table I, from which also the x-, ρ- and T-values at the limiting surface can be taken. Equation (20) or (21) then gives the corresponding u-value at the limiting surface. In order to obtain the value of

$$\int_0^x u^3 x^2 dx = -x^2 \frac{du}{dx} \tag{22}$$

[equation (19)] at the limiting surface, equation (5) must be considered. It follows that

$$M_r = 4\pi \left(\frac{1}{\pi G}\right)^{3/2} \frac{(\Re/\mu)^2}{(a/3)^{1/2}} \frac{(1-\beta')^{1/2}}{\beta'^2} \int_0^x u^3 x^2 dx,$$

or, according to equation (22),

$$M_r = 4\pi \left(\frac{1}{\pi G}\right)^{3/2} \frac{(\Re/\mu)^2}{(a/3)^{1/2}} \frac{(1-\beta')^{1/2}}{\beta'^2} \left(-x^2 \frac{du}{dx}\right). \tag{23}$$

This equation corresponds to Eddington's quartic equation, which is obtained, when $-x^2 \frac{du}{dx} = 2{\cdot}01824$ and $M_r = M$. Generally, in order to obtain the boundary value of $-x^2 \frac{du}{dx}$ which corresponds to the chosen $1-\beta'$, equation (23) must be solved by putting $M_r = M$.

The values of u^3, u and $-x^2 \frac{du}{dx}$ at the boundary surface, as found from equations (20), (21) and (23), are given in Table II. The corresponding x-values (Table I), rounded off to two decimals, are also included in the table.

Next write equation (2) in a form suitable for numerical calculation. Put $\frac{1}{3}aT^4$ according to (16) in the first member on the right, and ρ according to (16′) in the second member. As $1-\beta = 1-\beta'$, this leads to

$$L_r = -\frac{4\pi c(1-\beta')}{\kappa\rho} r^2 \frac{dP}{dr} + \frac{3}{4}\pi \frac{\beta'^2}{4-3\beta'} \left(\frac{3}{4} - \frac{1}{\gamma}\right) r^2 \frac{dP}{dr} ql,$$

or, by (4),

$$L_r = \frac{4\pi cGM_r(1-\beta')}{\kappa} - \frac{3}{4}\pi G\frac{\beta'^2}{4-3\beta'}\left(\frac{3}{4}-\frac{1}{\gamma}\right)M_r\rho q l.$$

Insert here ρ as given by (20). Then this equation can be written in the form

$$L_r = \frac{4\pi cGM(1-\beta')}{\kappa}\frac{M_r}{M} - \frac{3}{4}\left(\frac{1}{\pi G}\right)^{1/2}\frac{(\Re/\mu)^2 M(1-\beta')^{1/2}}{(a/3)^{1/2}R^2(4-3\beta')}\left(\frac{3}{4}-\frac{1}{\gamma}\right)q\frac{l}{R}u^3\frac{M_r}{M} \qquad (24)$$

$$= \frac{A}{\kappa}\frac{M_r}{M} - Bq\frac{l}{R}u^3\frac{M_r}{M}. \qquad (24A)$$

Here

$$\frac{M_r}{M} = \left(-x^2\frac{du}{dx}\right)\Big/\left(-x^2\frac{du}{dx}\right)_0 \qquad (25)$$

where the subscript zero refers to the outer boundary of the shell considered.

The factors, independent of x, in formula (24), namely the A and B of (24A), are

TABLE II

Initial Values at the Outer Boundary of the Shell with Combined Radiative and Turbulent Transfer

ϑ	x_0	u_0^3	u_0	$\left(-x^2\frac{du}{dx}\right)_0$
0·0	1·00	0·00	0·00	0·000
0·1	1·00	$7{\cdot}53.10^{-21}$	$1{\cdot}96.10^{-7}$	0·386
0·5	1·00	$5{\cdot}93.10^{-14}$	$3{\cdot}90.10^{-5}$	6·07
1·0	1·00	$1{\cdot}10.10^{-10}$	$4{\cdot}79.10^{-4}$	15·7
1·5	1·00	$1{\cdot}15.10^{-8}$	$2{\cdot}26.10^{-3}$	25·4
2	1·00	$3{\cdot}50.10^{-7}$	$7{\cdot}05.10^{-3}$	34·3
3	1·00	$4{\cdot}90.10^{-5}$	$3{\cdot}66.10^{-2}$	50·1
4	1·00	$1{\cdot}77.10^{-3}$	$1{\cdot}21.10^{-1}$	63·7
5	1·00	$3{\cdot}01.10^{-2}$	$3{\cdot}11.10^{-1}$	75·7
6	0·99	$3{\cdot}17.10^{-1}$	$6{\cdot}82.10^{-1}$	86·7
7	0·99	$2{\cdot}30.10^{0}$	$1{\cdot}32.10^{0}$	96·4
8	0·98	$1{\cdot}28.10^{1}$	$2{\cdot}34.10^{0}$	105
9	0·97	$5{\cdot}93.10^{1}$	$3{\cdot}90.10^{0}$	114
10	0·95	$2{\cdot}35.10^{2}$	$6{\cdot}17.10^{0}$	122

given in Table III in the second and third column. They have been first divided by the solar luminosity. These values must be understood in the following way: the envelope with purely radiative transfer reaches from the surface to a certain ϑ-value beneath the surface. Starting from this surface, the transfer is partly radiative (directed outwards) and partly turbulent (directed inwards). In this latter spherical shell the heat transfer occurs according to equation (24), and as the factors in question those must be used which correspond to the boundary-ϑ between this shell and the outer envelope.

In the fourth and fifth column of Table III are given the radiative flux of heat and the turbulent flux of heat divided by the product of the average turbulent speed and the mixing length, just at the boundary of both shells, at the side of the combined radiative and turbulent transfer. As the observed luminosity is $L/L_\odot = 1{\cdot}02.10^5$, which further is equal to $L_{\text{rad}}/L_\odot$ in the envelope with purely radiative transfer, it will be seen from the fourth column of Table III that, for any boundary surface between $\vartheta = 0$ and $\vartheta = 10$, the $(L_{\text{rad}}/L_\odot)$-value does not jump suddenly at the boundary surface. On the contrary, only small $q(l/R)$-values are needed (column 6) in order to counterbalance the excess

TABLE III

The Radiative and Turbulent Transfer at the Outer Boundary of the Turbulent Shell

1 ϑ	2 $A/L_\odot$	3 $B/L_\odot$	4 $\frac{L_{rad}}{L_\odot}$	5 $\frac{L_{turb}/L_\odot}{q(l/R)}$	6 $q(l/R)$
0·0	$5{\cdot}25.10^{6}$	0·00	$1{\cdot}00.10^{5}$	0·00	nearly zero
0·1	4·77	$7{\cdot}53.10^{1}$	0·97	$-5{\cdot}67.10^{-19}$	
0·5	3·50	$2{\cdot}99.10^{2}$	0·99	$-1{\cdot}77.10^{-11}$	
1·0	2·62	4·78	0·95	$-5{\cdot}26.10^{-8}$	
1·5	2·10	5·97	1·00	$-6{\cdot}87.10^{-6}$	
2	1·75	6·83	0·98	$-2{\cdot}39.10^{-4}$	
3	1·31	7·92	0·95	$-3{\cdot}88.10^{-2}$	
4	$1{\cdot}05.10^{6}$	8·55	0·91	$-1{\cdot}51.10^{0}$	
5	$8{\cdot}75.10^{5}$	8·93	0·92	$-2{\cdot}69.10^{1}$	
6	7·50	9·15	1·30	$-2{\cdot}90.10^{2}$	$9{\cdot}7.10^{1}$
7	6·56	9·26	1·37	$-2{\cdot}13.10^{3}$	$1{\cdot}6.10^{1}$
8	5·83	9·28	1·14	$-1{\cdot}19.10^{4}$	$1{\cdot}0.10^{0}$
9	5·25	9·29	1·17	$-5{\cdot}51.10^{4}$	$2{\cdot}7.10^{-1}$
10	$4{\cdot}77.10^{5}$	$9{\cdot}25.10^{2}$	$1{\cdot}44.10^{5}$	$-2{\cdot}17.10^{5}$	$1{\cdot}9.10^{-1}$

radiative transfer. As a matter of fact, the differences between $L_{obs}/L_\odot$ and $L_{rad}/L_\odot$ at the outer boundary of the shell, as caused by the jump in the temperature gradient *, seem in reality to be (absolutely taken) still smaller than is indicated by the values of Table III. First, the values of $L_{rad}/L_\odot$ from $\vartheta=0{\cdot}0$ to $\vartheta=5$ are smaller than the observed luminosity.† This seems to be caused by the fact that the envelope with purely radiative transfer was calculated without taking into account the scattering by free electrons. As is seen from Table I, the total opacity coefficient (κ) is somewhat larger than the photo-electric coefficient of opacity (κ_a). If the constitution of the envelope with purely radiative transfer had been determined according to the total opacity, L_{rad} from $\vartheta=0{\cdot}0$ to $\vartheta=5$ (Table III) apparently would have become very nearly equal to L_{obs}. At $\vartheta=6$ L_{rad} suddenly rises to a somewhat higher value. This comes from a sudden sinking of the photo-electric absorption at the corresponding temperature and density. Mathematically this appears as a fast increase of the guillotine factor which, when the constitution of the envelope with purely radiative transfer was calculated, was put equal to unity throughout.

In the outermost part of the shell with combined radiative-turbulent transfer of heat, the net flux must be practically equal to the observed luminosity. Therefore the $q(l/R)$-value in this part of the star is obtained by dividing $L_{rad}-L_{obs}$ by the quantity $-L_{turb}/q(l/R)$.‡ The values of L_{rad} from $\vartheta=0$ to $\vartheta=5$ (Table III) are practically equal to L_{obs}, and accordingly $q(l/R)$ is practically zero. Then, when a certain boundary surface is chosen (i.e. a certain ϑ-value), and the corresponding polytropic model is followed inwards, L_{rad} as well as $-L_{turb}/q(l/R)$ increase. As M_r is at first nearly constant, the former quantity changes nearly inversely proportionally to κ, while the latter quantity changes as u^3 [formula (24)]. κ is a combination of the photo-electric coefficient of absorption given by formula (9) and the coefficient of scattering given by formula (13). Let us now consider the guillotine factor τ as nearly constant and put

* Temperature, density and pressure, of course, change continuously.

† The accuracy of the calculation was such that the logarithms contained two decimals while the numerical values usually contained three significant figures.

‡ Generally, as will be shown later [formula (41)], such a calculation gives the maximum value of $q(l/R)$ at the point considered.

for ρ and T the expressions (20) and (21). Apart from a constant factor, formula (9) then assumes the form

$$\kappa_a \propto \sqrt{(u_0/u)}$$

where by u_0 is meant the u-value at the outer boundary of the shell. As is seen from Table I, the values of κ_a at the boundary surface, for all ϑ-values, are so big in comparison to κ_s($\log \kappa_s = 0{\cdot}40 - 1 = -0{\cdot}60$) that u may become several times bigger than u_0 without making κ_s, say, nearly equal to κ_a. (When $\vartheta = 10$, u must become about one hundred times greater than u_0 in order to make κ_a equal to κ_s. For smaller ϑ-values this limiting u is still greater.) As an approximation, we therefore neglect κ_s. Hence, $q(l/R)$ may be expressed approximately by the formula

$$q(l/R) = \{\sqrt{u_0/u}\,.(L_{\text{rad}})_0 - L_{\text{obs}}\} \Big/ \left[-\frac{(L_{\text{turb}})_0}{q(l/R)} \right] \left(\frac{u}{u_0} \right)^3$$

where L_{rad}, L_{obs} and L_{turb} may as well be expressed in solar units. The subscript zero, as before, refers to the outer boundary of the shell. If the ratio of the radiative flux of heat to the observed luminosity at the boundary is denoted by α, or if

$$\alpha = (L_{\text{rad}})_0 / L_{\text{obs}}, \tag{26}$$

the above formula can be expressed in the form

$$q(l/R) = \frac{L_{\text{obs}}}{-(L_{\text{turb}})_0/q(l/R)} \frac{\{\alpha\sqrt{u/u_0} - 1\}}{(u/u_0)^3}. \tag{27}$$

The quantity $(L_{\text{turb}})_0/[q(l/R)]$, divided by the solar luminosity, is given (for the different ϑ-values) in the fifth column of Table III.

Denote now $u/u_0 = z$ and call the second factor in formula (27) f. Accordingly

$$f = \frac{\alpha\sqrt{z} - 1}{z^3}, \tag{28}$$

$$\frac{df}{dz} = \frac{3 - \frac{5}{2}\alpha\sqrt{z}}{z^4} \tag{29}$$

and

$$\frac{d^2f}{dz^2} = \frac{\frac{35}{4}\alpha\sqrt{z} - 12}{z^5}. \tag{30}$$

Equation (29) shows that (for $u/u_0 = z > 0$) f reaches an extreme value at

$$\sqrt{(u/u_0)} = \sqrt{z} = \frac{6}{5}\frac{1}{\alpha}, \tag{31}$$

while equation (30) ensures that this extreme value is a maximum value ($d^2f/dz^2 < 0$).

From $\vartheta = 0$ to $\vartheta = 5$, α [formula (26); Table III] is practically equal to unity. Hence the value of u for which $q(l/R)$ (or f) is at the maximum, is greater than u_0 [(31)]. Accordingly this maximum value really occurs in the interior of these models. According to formulæ (28) and (31) $f_{\max} = 5^5/6^6 \approx 0{\cdot}0670$. Hence [(27)]

$$[q(l/R)]_{\max} = 0{\cdot}0670 \frac{L_{\text{obs}}}{-(L_{\text{turb}})_0/q(l/R)}, \tag{32}$$

in which formula the denominator is obtained from Table III. The $[q(l/R)]_{\max}$-values thus found are given in Table IV. Stress is laid on the fact that these values represent the really greatest values of $q(l/R)$ in the interior of these models and not only the greatest possible ones.

For the ϑ-values from 6 to 10, α is equal to 1·28, 1·34, 1·12, 1·15 and 1·41 respectively. Hence in these cases, except when $\vartheta=8$ and $\vartheta=9$, the u-value which corresponds to the maximum $q(l/R)$ is smaller than u_0 [formula (31), $6/5=1{\cdot}2$]. Hence these $q(l/R)$-values do not occur at all in the interior of the corresponding models. Then the greatest value of $q(l/R)$ is the value at the boundary surface and may be taken from Table III. These values are added to Table IV.

TABLE IV

Maximum Values of $q(l/R)$ for Models started with Different ϑ-Values

ϑ	$\dfrac{L_{obs}}{-(L_{turb})_0/q(l/R)}$	$[q(l/R)]_{max}$	If $l/R=1/1000$ q_{max} (cm./sec.)
0·0	∞	∞	∞
0·1	$1{\cdot}80.10^{23}$	$1{\cdot}2.10^{22}$	$1{\cdot}2.10^{25}$
0·5	$5{\cdot}76.10^{15}$	$3{\cdot}9.10^{14}$	$3{\cdot}9.10^{17}$
1·0	$1{\cdot}94.10^{12}$	$1{\cdot}3.10^{11}$	$1{\cdot}3.10^{14}$
1·5	$1{\cdot}48.10^{10}$	$9{\cdot}9.10^{8}$	$9{\cdot}9.10^{11}$
2	$4{\cdot}26.10^{8}$	$2{\cdot}9.10^{7}$	$2{\cdot}9.10^{10}$
3	$2{\cdot}63.10^{6}$	$1{\cdot}8.10^{5}$	$1{\cdot}8.10^{8}$
4	$6{\cdot}75.10^{4}$	$4{\cdot}5.10^{3}$	$4{\cdot}5.10^{6}$
5	$3{\cdot}79.10^{3}$	$2{\cdot}5.10^{2}$	$2{\cdot}5.10^{5}$
6	$3{\cdot}51.10^{2}$	$9{\cdot}7.10^{1}$	$9{\cdot}7.10^{4}$
7	$4{\cdot}79.10^{1}$	$1{\cdot}6.10^{1}$	$1{\cdot}6.10^{4}$
8	$8{\cdot}57.10^{0}$	$1{\cdot}1.10^{0}$	$1{\cdot}1.10^{3}$
9	$1{\cdot}85.10^{0}$	$2{\cdot}9.10^{-1}$	$2{\cdot}9.10^{2}$
10	$4{\cdot}70.10^{-1}$	$1{\cdot}9.10^{-1}$	$1{\cdot}9.10^{2}$

When $\vartheta=8$, $\alpha=1{\cdot}12$ and $\sqrt{(u/u_0)}=\sqrt{z}=1{\cdot}2/1{\cdot}12=1{\cdot}07$. The $q(l/R)_{max}$-value given by equation (32) must be multiplied by $\alpha^6=1{\cdot}97$. When $\vartheta=9$, $\alpha=1{\cdot}15$ and $\sqrt{(u/u_0)}=\sqrt{z}=1{\cdot}2/1{\cdot}15=1{\cdot}04$. Further $\alpha^6=2{\cdot}31$. The $q(l/R)$-values thus found are added to Table IV.

One gets a better understanding of the meaning of the size of $[q(l/R)]_{max}$ if the latter is divided by a reasonable value assumed for l/R. A maximum reasonable value of l/R seems to be 1/1000. Longer mixing lengths must seem rather long. On the other hand there appears to be no evidence for the non-existence of much shorter mixing lengths. Hence, when the $[q(l/R)]_{max}$-values are divided by 1/1000 (i.e. multiplied by 1000), the order of magnitude of the smallest possible average speed is obtained, at the place where the speed is at the maximum. These values of q are added to Table IV. It is evident that the values at the upper end of the table must be rejected as being too high. At the lower end, however, they must be considered as low values for a turbulent velocity. In order to avoid very high concentrations of the mass towards the surface it seems sensible to choose the model with $\vartheta=6$ as the base for further calculations. In this model the maximum value of q (about 10^5 cm./sec. = 1000 m./sec.) must be considered as a rather high average speed of the turbulence. It is, however, much below the average turbulent speed, 18·4 km./sec., suggested for the solar chromosphere.* Much lower speeds, only $2{\cdot}10^3$ cm./sec. = 20 m./sec., are shown by the model $\vartheta=8$, although this model does not show a concentration of the mass towards the surface much higher than the model $\vartheta=6$.

The equation (19) was now integrated numerically, with the initial values of x, u and $-x^2\dfrac{du}{dx}$ as taken from Table II. The integration was carried out with the step

* Unsöld, *Ap. J.*, 69, 218, 1929; McCrea, *M.N.*, 89, 724, 1929.

0·01 and the number of decimals used in u was three. The result of the calculation is given in Table V. However, the values are given to that depth in the stellar interior only, to which the "polytropic" solution was used in the final model. M_r/M was calculated from equation (25) and ρ and T from (20) and (21) respectively. κ is found as in section 3. Then the quantities L_{rad} and $L_{\text{turb}}/[q(l/R)]$ were calculated from equation (24) and divided by $L_\odot$ (Table V, columns 9 and 10). How certain maximum values of $q(l/R)$ may be deduced from the two latter quantities will be explained in section 7.

5. *The Inner Part of the Star with an Outwards Directed Turbulent Transfer.*—It will be seen from the polytropic solution, considered in the preceding section, that the density rises very fast in the outer part of the star.* The whole mass of the star will soon be used up by the "polytropic" shell. If the solution of (19) is followed still deeper the mass and density will become negative. This fact seems to prove that the interior of NGC 6871,5 must be more or less homogeneous.† Probably the density has a minimum at the stellar centre. This can be understood as follows: the large-scale currents, especially the speed of rotation varying with distance from centre, create turbulence. This turbulence may then practically dissolve radial convection currents which tend to alter the density distribution. (The latter phenomenon may also be interpreted as caused by a sufficiently high turbulent viscosity.)

In order to make the interior as simple as possible for the mathematical treatment, a constant density will now be chosen for the entire part of the star inside the "polytropic" shell. In the latter shell the density is proportional to the quantity u^3, while the mass is proportional to the integral

$$\int_0^x u^3 x^2 dx. \tag{33}$$

The density, as well as the mass, are continuous at the boundary surface. It is convenient to choose a constant number u^3 so that it, multiplied by the same factor as u^3 in the polytropic shell [formula (20)], gives the density. Then the integral (33), multiplied by the same factor as the same integral $\left(\text{or the quantity } -x^2\frac{du}{dx}\right)$ in the polytropic shell [formula (23)], gives the mass. From the continuity of the density and mass at the boundary surface it then follows that the solution of the equation

$$u^3 = -\frac{3}{x}\frac{du}{dx} \tag{34}$$

for the polytropic shell gives the value of u^3 in the core. The solution of equation (34) is easily found by interpolation from the scheme of calculation. The ρ-value in the core is then found from equation (20).

In order to find the T-values, the pressure at each point of the core must first be calculated. The latter is found by integration from equations (4) and (5). If at the limiting surface the distance from the centre is denoted by x_b and the total pressure by P_b, it then follows that the pressure in the core is given by

$$P = P_b + \tfrac{2}{3}\pi G R^2 \rho_c^2 (x_b^2 - x^2), \tag{35}$$

---

* If the distribution of the density corresponded to a purely radiative transfer, the density would rise still faster. (Compare p. 261.)

† Chandrasekhar, based on investigation of purely radiative transfer in the outer part of the stars, already stated that Trumpler's stars must be very much more homogeneous than any of the other known stars (*M.N.*, 96, 658, 1936). Chandrasekhar's investigations, however, do not explain, for instance, how a star can be more homogeneous than the adiabatic model. As already stated in the present paper (p. 261), if the solution with purely radiative transfer is continued sufficiently far into the interior, the density will finally become negative.

where ρ_c is the density in the core. When P is calculated from this equation, the temperature T can be solved from equation (3). Then

$$1-\beta=\frac{\frac{1}{3}aT^4}{P}. \tag{35'}$$

The next task is to express the equation of transfer [(2)] in a form suitable for the core. Equation (2) is first simplified by the fact that $d\rho/dr=0$. Secondly, from (3), or

$$\tfrac{1}{3}aT^4=P-\frac{\Re}{\mu}\rho_c T,$$

it is easily found that

$$\frac{d(\frac{1}{3}aT^4)}{dr}=4\frac{1-\beta}{4-3\beta}\frac{dP}{dr}.$$

Accordingly relation (2) is reduced to the form

$$L_r=-16\pi c\frac{r^2}{\kappa\rho_c}\frac{1-\beta}{4-3\beta}\frac{dP}{dr}-\frac{3}{4}\pi r^2\frac{\beta^2}{(4-3\beta)\gamma}\frac{dP}{dr}ql,$$

or, according to (4),

$$L_r=\frac{16\pi cG}{\kappa}\frac{1-\beta}{4-3\beta}M_r+\frac{3}{4}\pi G\rho_c\frac{\beta^2}{(4-3\beta)\gamma}M_r ql.$$

It is suitable to insert for M_r the expression obtained from (5),

$$M_r=\tfrac{4}{3}\pi R^3\rho_c x^3. \tag{36}$$

Further, instead of l the ratio l/R will be used. Then the above expression of L_r takes the form

$$L_r=\frac{64\pi^2cG}{3\kappa}R^3\rho_c\frac{1-\beta}{4-3\beta}x^3+\pi^2GR^4\rho_c^2\frac{\beta^2}{(4-3\beta)\gamma}x^3q(l/R). \tag{37}$$

Here the values of $1-\beta$ and γ are found from equations (35') and (15). For κ see section 3. The values of L_{rad} and $L_{\text{turb}}/[q(l/R)]$, thus found, divided by the solar luminosity, are given in Table V. The values of u^3, the relative mass, P, ρ, T, $1-\beta$ and κ are also included in the table. The relative mass is found from equation (36) when it is divided by the total mass M.

6. *The Fitting of the Envelope, the Polytropic Shell and the Core to one another.*— Table V gives the constitution of NGC 6871,5 as represented by the envelope with purely radiative transfer *, the polytropic shell, and the core with a constant density. As will be seen from the table, there is a slight jump in the radiative flux of heat at the place where the envelope changes to the polytropic shell. This jump may even be only apparent. (Compare p. 264.) On the other hand, the turbulent flux, divided by the product ql, shows a similar insignificant jump at the same place.

A more pronounced jump in the radiative flux of heat occurs at $x=x_b$ where the polytropic shell changes to the core with a constant density. At the same place the turbulent flux of heat, divided by ql, jumps from a certain negative value to a certain positive value.

Let us now imagine slight adjustments in the density distribution curve, so that the radiative flux of heat L_{rad} as well as the quantity $L_{\text{turb}}/(ql)$ will vary continuously. The imagined alteration in the density curve can be made so small that the alterations in ρ, T and P may be neglected in the calculation of L_{rad} and $L_{\text{turb}}/(ql)$. Then, as is seen from equation (2), L_{rad} is altered proportionally to dT/dr. On the other hand, as is seen from equation (4), dP/dr remains practically unaltered. Hence, according to equation (2), the alteration of $L_{\text{turb}}/(ql)$ is proportional to the alteration of $d\rho/dr$. When

* The constitution of this part of the star is given in greater detail in Table I.

TABLE V

A Model for Trumpler's Star NGC 6871,5 which can be made physically understandable

1	2	3	4	5	6	7	8	9	10
x	u^3	M_r/M	$P.10^{-14}$	ρ	$T.10^{-6}$	$1-\beta$	κ	$L_{rad}/L_\odot$	$\frac{L_{turb}/L_\odot}{q(l/R)}$
1·00	0·000	1·0000	0·00	0·000000	0·00	1·000	52·5	1·02.10⁵	0·0
0·99	0·317	1·0000		·000149	0·93	0·143	5·75	1·02	−3·3.10²
0·99	0·317	1·0000		·000149	0·93	·143	5·75	1·30.10⁵	−2·90.10²
0·98	3·914	0·9998		·00184	2·14	·143	3·98	1·88	−3·58.10³
0·97	15·40	·9989		·00722	3·38	·143	3·63	2·06	−1·41.10⁴
0·96	39·90	·9960		·0187	4·65	·143	2·95	2·53	−3·64.10⁴
0·95	82·88	·9898		·0389	5·93	·143	2·29	3·24	−7·51.10⁴
0·94	150·3	·9780		·0705	7·23	·143	1·78	4·12	−1·35.10⁵
0·93	247·3	·9582		·116	8·54	·143	1·41	5·10	−2·17.10⁵
x_b*	309·8	·9434		·145	9·25	·143	1·23	5·76.10⁵	−2·67.10⁵
x_b	309·8	·9434	1·30	·145	9·25	·143	1·23	15·6.10⁵	+17·2.10⁵
0·9	309·8	·8683	3·11	·145	15·1	·420	0·447	74·2	+48·1.10⁴
0·8	309·8	·6098	9·79	·145	23·0	·719	·309	92·4	+59·3.10³
0·7	309·8	·4085	15·69	·145	26·5	·797	·288	68·6	+19·5.10³
0·6	309·8	·2573	20·80	·145	28·8	·834	·276	45·6	+79·9.10²
0·5	309·8	·1489	25·12	·145	30·4	·855	·269	27·3	+34·8
0·4	309·8	·0762	28·66	·145	31·5	·868	·269	14·0	+14·6
0·3	309·8	·0322	31·41	·145	32·3	·876	·269	5·9	+ 5·4
0·2	309·8	·0095	33·37	·145	32·9	·882	·263	1·8	+ 1·4.10²
0·1	309·8	·0012	34·55	·145	33·2	·885	·263	0·2.10⁵	+ 1·7.10¹
0·0	309·8	0·0000	34·94	0·145	33·3	0·886	0·263	0·0	0·0

dT/dr and $d\rho/dr$ are altered, they must be altered in such a way that the equation (3) remains satisfied. Accordingly the equation

$$\frac{d\rho}{dr}=\left[\frac{dP}{dr}-\left(\frac{\Re}{\mu}\rho+4.\tfrac{1}{3}aT^3\right)\frac{dT}{dr}\right]\Big/\frac{\Re}{\mu}T,$$

obtained from (3) by differentiation, must be satisfied. The latter relation shows that the alteration of $d\rho/dr$ is proportional to the alteration of dT/dr. Hence the alteration of L_{rad}, as well as the alteration of $L_{turb}/(ql)$, are proportional to the alteration of dT/dr. Accordingly, when a weighted mean of the two L_{rad}-values at a limiting surface is built, the corresponding $L_{turb}/(ql)$-value is obtained by building, with the same weights, a mean value of the two $L_{turb}/(ql)$-values. At $x=x_b$, it is then natural to calculate the average value so that $L_{turb}/(ql)$ vanishes. The turbulent flux of heat at $x=x_b$ is then zero for any ql-value. At the limiting surface of the envelope and the polytropic shell, it is natural to round off the density curve by taking a straight mean of the two values of L_{rad} and $L_{turb}/(ql)$ at the limiting surface.

The values of the radiative flux of heat, and of the turbulent flux of heat divided by $q(l/R)$, for the model thus adjusted, are given in Table VI.

7. *The Sources and Sinks of Heat.*—As is seen from Table VI, the turbulent flux of heat is positive from the stellar centre to $x=x_b$, and negative from $x=x_b$ outwards. In the vicinity of the stellar surface the turbulent flux vanishes. Accordingly the L_{rad}-values in the inner part of the star represent the minimum total flux of heat, while in the outer part of the star they represent the maximum total flux of heat. At $x=x_b$,

* $x_b=0{\cdot}92524$.

TABLE VI

The Flux of Heat in NGC 6871,5

x	M_r/M	$L_{\text{rad}}/L_\odot$	$\frac{L_{\text{turb}}/L_\odot}{q(l/R)}$	$q(l/R) \leqslant$
1·00	1·0000	$1{\cdot}02.10^5$	0·00	0
0·99	1·0000	1·16	$-\ 3{\cdot}10.10^2$	45
0·98	0·9998	1·88	$-\ 3{\cdot}58.10^3$	24
0·97	·9989	2·06	$-\ 1{\cdot}41.10^4$	7·4
0·96	·9960	2·53	$-\ 3{\cdot}64.10^4$	4·2
0·95	·9898	3·24	$-\ 7{\cdot}51.10^4$	3·0
0·94	·9780	4·12	$-\ 1{\cdot}35.10^5$	2·3
0·93	·9582	5·10	$-\ 2{\cdot}17.10^5$	1·9
x_b	·9434	7·08	0·00	
0·9	·8683	74·2	$+48{\cdot}1.10^4$	
0·8	·6098	92·4	$+59{\cdot}3.10^3$	
0·7	·4085	68·6	$+19{\cdot}5.10^3$	
0·6	·2573	45·6	$+79{\cdot}9.10^2$	
0·5	·1489	27·3	$+34{\cdot}8.10^2$	
0·4	·0762	14·0	$+14{\cdot}6.10^2$	
0·3	·0322	5·9	$+\ 5{\cdot}4.10^2$	
0·2	·0095	1·8	$+\ 1{\cdot}4.10^2$	
0·1	·0012	$0{\cdot}2.10^5$	$+\ 1{\cdot}7.10^1$	
0·0	0·0000	0·0	0·0	

where the flux of heat is purely radiative, it is about seven times the stellar luminosity. At $x = 0{\cdot}8$, where the radiative flux of heat reaches a maximum, the total flux is at least $92{\cdot}4.10^5$ solar luminosities or about 90 times the stellar luminosity. Such a high flux of heat seems to be inevitable somewhere in the star. The Eddington luminosity formula [(I)] gives, for $\mu = 1$, the value $66{\cdot}8.10^5$ solar luminosities, which is only slightly less than the above maximum value. The total flux of heat increases from the centre, reaches a maximum somewhere in the star, and then drops again towards the surface. Such a state of affairs can be understood if there are strong sources of heat in the inner part of the star and sinks of heat in the outer part of the star.

As was shown in (III) [(III), formula (6) combined with (4)], the total generation of heat within a sphere of radius r can be expressed by the formula

$$L_r = 4\pi \int_0^r \left[\epsilon_s + \frac{1}{2} \frac{q^3}{l} - \frac{P}{\rho r^2} \frac{d(r^2 \bar{U})}{dr} - \epsilon_g \right] \rho r^2 dr. \tag{38}$$

Here the first term in the brackets, ϵ_s, is the generation of subatomic heat, per unit mass and time. The second term represents the generation of heat, per unit mass and time, when turbulence is dissipated by friction. The third term, without the minus sign, gives the net amount of heat (positive or negative) which, per unit mass and time, is changed into mechanical work when the gas, due to the turbulence, expands and contracts in irregular succession. Finally, ϵ_g is the amount of heat which, per unit mass and time, is changed into kinetic energy of the large-scale motions.

Generally, when the integral in (38) is calculated, the value obtained for L_r must be equal to the total flux of heat for the same r-value. Consider now equation (38) at the place where L_r is at the maximum (Table VI), i.e. at least $92{\cdot}4.10^5$ solar luminosities or 90 times the observed luminosity of the star. The fraction of the radius up to this distance from the centre is 0·8 and the fraction of the mass 0·6098. As is well known,

practically all the subatomic heat is generated in the highest temperatures of a star.* Therefore we are justified in writing

$$\bar{\epsilon}_s = \frac{L}{0{\cdot}6098M},$$

where the bar refers to an average value per unit mass. Hence it is found that

$$\bar{\epsilon}_s = 800.$$

ϵ_g can be only positive [(III), p. 171] or, locally in the star, zero. It follows then that heat is converted into kinetic energy of large-scale motions mostly in the outer part of the star. Therefore we put ϵ_g from centre to $x = 0{\cdot}8$ equal to zero. Further, we confine ourselves to such values of $q(l/R)$ only which make the turbulent flux, up to $x = 0{\cdot}8$, practically negligible in comparison with the radiative flux. As is seen from Table VI, values as high as $q(l/R) = 100$ satisfy this condition. Then, at $x = 0{\cdot}8$, the total flux of heat is nearly equal to the radiative flux of heat or about 90 times the heat generated by subatomic processes. In order that L_r at $x = 0{\cdot}8$ be 90 times the observed luminosity, it is then necessary that [formula (38)]

$$\text{mean value of } \left(\frac{1}{2}\frac{q^3}{l} - \frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr}\right) \approx 90\bar{\epsilon}_s \tag{39}$$

where

$$90\bar{\epsilon}_s \approx 7{\cdot}2 \,.\, 10^4. \tag{39'}$$

Let us now see which values of q and l satisfy equation (39). First estimate the second term on the left. Because of the constancy of ρ in the core, it first follows from equation (1')

$$\bar{U} = -\frac{1}{8}ql\frac{1}{\gamma P}\frac{dP}{dr},$$

or, according to (4),

$$r^2\bar{U} = \frac{1}{8}Gql\frac{1}{\gamma P}M_r\rho_c.$$

Further, according to (5),

$$M_r = \frac{4\pi}{3}\rho_c r^3.$$

Hence

$$r^2\bar{U} = \frac{\pi G}{6}\rho_c{}^2\frac{ql}{\gamma}\frac{r^3}{P}.$$

The quantity γ is nearly constant. Hence, as an approximation the expression ql/γ as well as ql may be considered as constant. Then it follows that

$$\frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr} = \frac{\pi G}{6}\rho_c\frac{ql}{\gamma}\left(3 - \frac{r}{P}\frac{dP}{dr}\right),$$

which may be rewritten similarly to the quantity $\bar{U}$ above. Hence,

$$\frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr} = \frac{\pi G}{6}\rho_c\frac{ql}{\gamma}\left(3 + \frac{4\pi G}{3}\rho_c{}^2\frac{r^2}{P}\right).$$

* If Bethe's theory (*Phys. Rev.*, 55, 448–452, 1939) of liberation of subatomic heat is applied to NGC 6871,5, assuming it to have the standard model constitution, the heat generated in the star will be $2{\cdot}1 \,.\, 10^9$ solar luminosities. The corresponding value for the polytropic model of index $n = 1{\cdot}5$ will be $3{\cdot}8 \,.\, 10^6$. The model treated in the present paper gives the value about $1{\cdot}0 \,.\, 10^4$, while the model with a constant density gives $4{\cdot}7 \,.\, 10^3$. The observed value is $L_{obs} = 1{\cdot}02 \,.\, 10^5 L_\odot$ which corresponds to a model about half-way between the polytropic model of index $n = 1{\cdot}5$ and the model calculated in the present paper.

The second term in the parenthesis is equal to ($\rho_c = 0{\cdot}145$, Table V)

$$\frac{4\pi G}{3}\rho_c^2 R^2 \frac{x^2}{P} = 7{\cdot}80 \,.\, 10^{15}\frac{x^2}{P}.$$

In the stellar centre the latter expression is equal to zero, while the values corresponding to the x-values

$$0{\cdot}8, \qquad 0{\cdot}7, \qquad 0{\cdot}6$$

are (as calculated from Table V)

$$5{\cdot}10, \qquad 2{\cdot}43, \qquad 1{\cdot}35$$

respectively, and the corresponding relative masses

$$0{\cdot}6098, \qquad 0{\cdot}4085, \qquad 0{\cdot}2573$$

respectively. The mean of 5·10 and zero is 2·55, while the value corresponding to a half of the mass within $x = 0{\cdot}8$ is about 1·69. It seems that the value 2 may be considered as a sufficiently accurate average value of $7{\cdot}80 \,.\, 10^{15} x^2/P$. Then

$$\text{mean value of } \frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr} \approx \frac{5\pi G}{6}\rho_c \frac{R}{\gamma} q(l/R).$$

For the $(1-\beta)$-values in question (Table V), $1/\gamma$ [formula (15)] is on the average about 0·73. Hence

$$\text{mean value of } \frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr} \approx 2{\cdot}1 \,.\, 10^4 q(l/R).$$

Taking into account the latter formula and formula (39′), the relation (39) may, finally, be rewritten in the form

$$4{\cdot}3 \,.\, 10^{-13}\frac{[q(l/R)]^3}{(l/R)^4} - 2{\cdot}1 \,.\, 10^4 q(l/R) \approx 7{\cdot}2 \,.\, 10^4.$$

This can be solved for l/R, whence

$$(l/R)^4 \approx \frac{[q(l/R)]^3 \,.\, 10^{-16}}{4{\cdot}9 q(l/R) + 16{\cdot}6}. \tag{40}$$

From this formula one can calculate the value of l/R which corresponds to a given value of $q(l/R)$. When the latter value is divided by the given value of l/R, the corresponding value of q is obtained.

Formula (40) has been applied in Table VII. It will be seen from this table that such high values of $q(l/R)$ which yet do not considerably increase the total flux from

TABLE VII

Sources and Sinks of Heat in the Core up to $x = 0{\cdot}8$ for Different Values assumed for $q(l/R)$

$q(l/R)$	l/R	q (cm./sec.)	Mean Values of		$\bar{\epsilon}_g$	ϵ_g
			$\frac{1}{2}\frac{q^3}{l}$	$\frac{P}{\rho r^2}\frac{d(r^2\bar{U})}{dr}$		
0·01	$1{\cdot}6 \,.\, 10^{-6}$	$6{\cdot}4 \,.\, 10^3$	$7{\cdot}2 \,.\, 10^4$	$2{\cdot}1 \,.\, 10^2$	$8{\cdot}0 \,.\, 10^2$	0
0·1	$8{\cdot}8 \,.\, 10^{-6}$	$1{\cdot}1 \,.\, 10^4$	$7{\cdot}4 \,.\, 10^4$	$2{\cdot}1 \,.\, 10^3$	$8{\cdot}0 \,.\, 10^2$	0
1	$4{\cdot}6 \,.\, 10^{-5}$	$2{\cdot}2 \,.\, 10^4$	$9{\cdot}4 \,.\, 10^4$	$2{\cdot}1 \,.\, 10^4$	$8{\cdot}0 \,.\, 10^2$	0
10	$2{\cdot}0 \,.\, 10^{-4}$	$5{\cdot}1 \,.\, 10^4$	$2{\cdot}9 \,.\, 10^5$	$2{\cdot}1 \,.\, 10^5$	$8{\cdot}0 \,.\, 10^2$	0
100	$6{\cdot}6 \,.\, 10^{-4}$	$1{\cdot}5 \,.\, 10^5$	$2{\cdot}2 \,.\, 10^6$	$2{\cdot}1 \,.\, 10^6$	$8{\cdot}0 \,.\, 10^2$	0

the radiative flux value, nevertheless make the average turbulent speeds rather high. *Vice versa*, even high values of the turbulent speed do not need to increase greatly the luminosity. Further, it will be seen from the table that the generation of heat by dissipation of turbulence by friction is much greater than the liberation of subatomic heat.

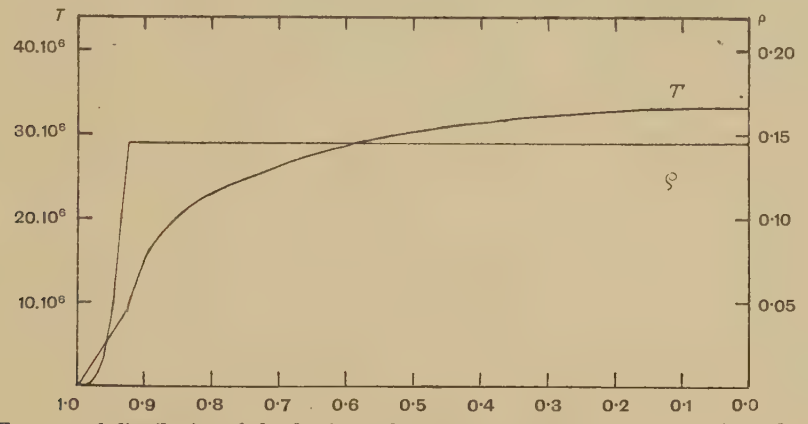


FIG. 1.—*A distribution of the density and temperature in NGC 6871,5 which can be made physically understandable and is in agreement with the observed mass, luminosity and radius. (Abscissæ : distance from centre.)*

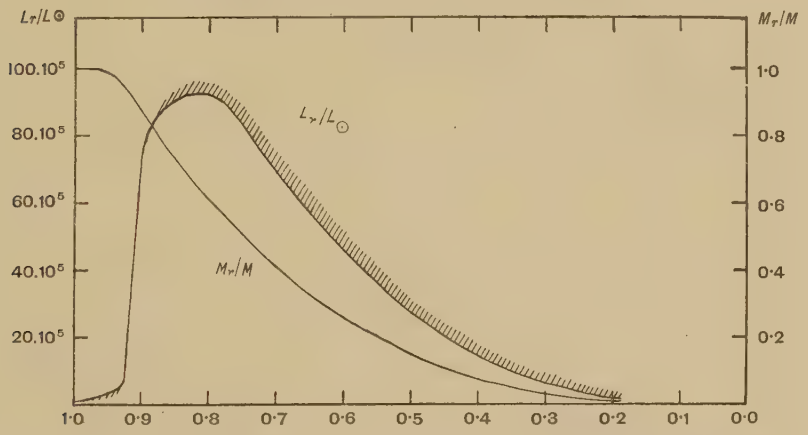


FIG. 2.—*The corresponding relative mass and flux of heat. For the flux of heat, either a lower limit (in the inner part) or an upper limit (in the outer part) is calculable.*

At distances greater than $x = 0{\cdot}8$ from the centre, the value of ϵ_g must increase rapidly, so that the expression in brackets in (38) becomes negative. It is then natural to assume that the net flux of heat diminishes continuously when the surface is approached. Then

$$L_r = L_{\text{obs}} + L',$$

where $L' > 0$ (except in the vicinity of the surface where $L' = 0$). On the other hand,

$$L_r = L_{\text{rad}} - \frac{-L_{\text{turb}}}{q(l/R)} \cdot q(l/R),$$

where $-L_{\text{turb}} > 0$. When L_r is eliminated from the two last equations, it follows that

$$q(l/R) = \frac{L_{\text{rad}} - L_{\text{obs}}}{-L_{\text{turb}}/q(l/R)} - \frac{L'}{-L_{\text{turb}}/q(l/R)}.$$

Hence

$$q(l/R) \leqslant \frac{L_{\text{rad}} - L_{\text{obs}}}{-L_{\text{turb}}/q(l/R)}. \tag{41}$$

The upper limits of $q(l/R)$ found from formula (41) are added to Table VI. The sign of equality is valid in the vicinity of the surface ($x = 1$). In the vicinity of $x = 0{\cdot}93$ the $q(l/R)$-values must be nearly zero. The biggest values of $q(l/R)$ occur in the vicinity of the surface. The question of the magnitude of the values of q and l separately can be discussed similarly as in section 4 (p. 266). However, we may content ourselves with the said discussion, because the biggest values of $q(l/R)$ occur just at the places with which the latter discussion deals and the only values of q and l which we try to avoid are the big ones.

The quantity ϵ_g [formula (38)], the rate of conversion of heat into kinetic energy of large-scale currents, will not be treated any further in the present paper. Actually it should be considered in connection with the large-scale currents in rotating stars. Another quantity involved in the suggested theory of Trumpler's stars is the rate of conversion of the kinetic energy of the large-scale currents into kinetic energy of the turbulence, ϵ_t. This quantity appears in the formula [(III), formulæ (4) and (5)]

$$\frac{1}{r^2}\frac{d(\frac{1}{2}\rho q^2 . r^2 \bar{U})}{dr} = -\frac{dP}{dr}\bar{U} - \frac{1}{2}\rho\frac{q^3}{l} + \rho\epsilon_t. \tag{42}$$

The distribution of the density and temperature in the model calculated has been represented graphically in Fig. 1. The relative mass and the flux of heat, as a function of the distance from the centre, are given in Fig. 2.

Astronomical Observatory,
Helsinki:
1945 *March.*

THE DEEP MINIMUM IN THE LIGHT-CURVE OF NOVA HERCULIS 1934

F. J. M. Stratton

(Received 1945 November 2)

Summary

The deep minimum in the light-curve, which appeared in the transition stage of Nova Herculis 1934 and Nova Cygni 1942, was accompanied in both cases by the weakening of the continuous spectrum, the suppression of the absorption spectrum and of the bright bands of *C*II, *N*II, *O*II emitted by a shell of late origin. The violet maxima of the bright bands of *H*, *Fe*II, *Ca*II from the earlier or outer shell strengthened and moved out to cover the positions previously occupied by the corresponding absorption lines; the red maxima weakened or vanished.

An explanation is offered in terms of an opaque cloud forming inside the outer shell passing across the central star, obscuring it and the inner shells. This implies that the later shells do not move out with the velocities of the spectroscopically observable atoms that traverse them. Similar conclusions were reached by Tai in the case of Nova Herculis from a study of the contours of lines showing double absorptions. A study of Nova Persei 1901 also supports the view that the speed of outward spread of visible shells is less than the velocity of atoms passing through them. Comparison is made with changes of spectrum found at the minima in the stage of fluctuating luminosity of fast novæ.

The source of the surviving continuous spectrum is uncertain. The view that it is due to reflected light from portions of the outer shell still illuminated by the central star gains support from the spectrum of Hubble's variable nebula. No final explanation is offered on the nature of the obscuring cloud, but some connection with the outflowing atoms of carbon, nitrogen and hydrogen is suggested.

---

1. *History of the Minimum.*—Nova Herculis 1934 was a typical slow nova: it reached its maximum brightness, $1^{m}\cdot3$, on 1934 December 23, ten days after discovery, and then faded with minor fluctuations at the rate of one magnitude a month. On 1935 March 31, when its magnitude was $4^{m}\cdot4$, a rapid decline in brightness began at the rate of nearly one magnitude a day for the first four days, continuing at a slower rate to a minimum of $13^{m}\cdot4$ on May 3. From that date the star rose in brightness again to $7^{m}\cdot0$ in early July, oscillating slightly about that magnitude for the rest of the year. Two marked changes in spectrum accompanied the minimum: the first, the subject of this note, took place between April 3 and 5; the second took place during the brightening of the star after the deep minimum and was marked by the emergence of the lines of [*O*III] at 4363 A., 4959 A. and 5007 A., and of other bright lines typical of planetary nebulæ. This second change is parallel to the observed changes after the deep minima of Nova Aurigæ 1891 and Nova Cygni 1942.

2. *The Complex Spectrum before the Minimum.*—In describing the spectrum before the fall of brightness it is convenient to make use of the notation employed by McLaughlin

in his careful and full study of the star's spectrum.* Successive spectra of absorption and emission have been numbered by him. Those surviving at the end of March 1935 were the following:—

Absorption II. An α Cygni spectrum with a displacement to the violet due to a line-of-sight velocity which had increased from about −320 km./sec. on 1934 December 26 to about −400 km./sec. at the end of March 1935.

Emission II. Accompanying undisplaced emission bands formed the central and brighter portions of the bright band spectrum of *H*, *Ca*II; two bright maxima were seen in the bands of *H*, *Ca*II, *Fe*II, with velocity displacements to the violet and red less than the displacements to the violet of Absorption II. Lines of [*O*I], [*N*II], and later of [*Fe*II] also showed these double maxima, with separations slightly higher than those of the permitted lines.†

Absorptions III and IV. Two additional α Cygni spectra with higher velocity displacements. By the end of March these only survived in the hydrogen lines with approximate displacements of −700 and −800 km./sec. respectively.

Emissions III and IV. These broader accompanying emissions had faded away by the end of March except for the hydrogen lines, where they were merged in still wider bands.

Absorption V. A set of fine components of the hydrogen lines with velocity displacements about −1000 km./sec. The presence of accompanying emission is uncertain.

Absorption VI. A set of B-type lines due to *C*II, *N*II, *O*II and *He*I with velocity displacements falling from about −600 km./sec. in early March nearly to −400 km./sec. at the end of the month. Hydrogen lines, if present, blend with other absorption components.

Emission VI. Strong emission bands of *C*II, *N*II, *O*II and *He*I accompanied Absorption VI. Their widths were uncertain owing to blends, to overlapping, and to the wider bands of Emission VIII in which they were merged.

Absorption VII. A set of lines due to *H* and *Fe*II intermediate between Absorptions II and IV and possibly a survival of Absorption III. There are no obvious associated emissions.

Absorption VIII. A set of absorption lines of *C*II, *N*II, *O*II and *He*I with velocity displacements between −950 and −1100 km./sec. The hydrogen lines, if present, would be merged with Absorption V on some days. On the whole the evidence is against their presence.

Emission VIII. Bright undisplaced bands accompanying Absorption VIII.

Absorption XI. A set of diffuse absorption lines of hydrogen with velocity displacements varying between −1200 and −1700 km./sec.

Emission XI. Hydrogen undisplaced bright bands extending at the red end to an amount comparable with the absorption velocities.

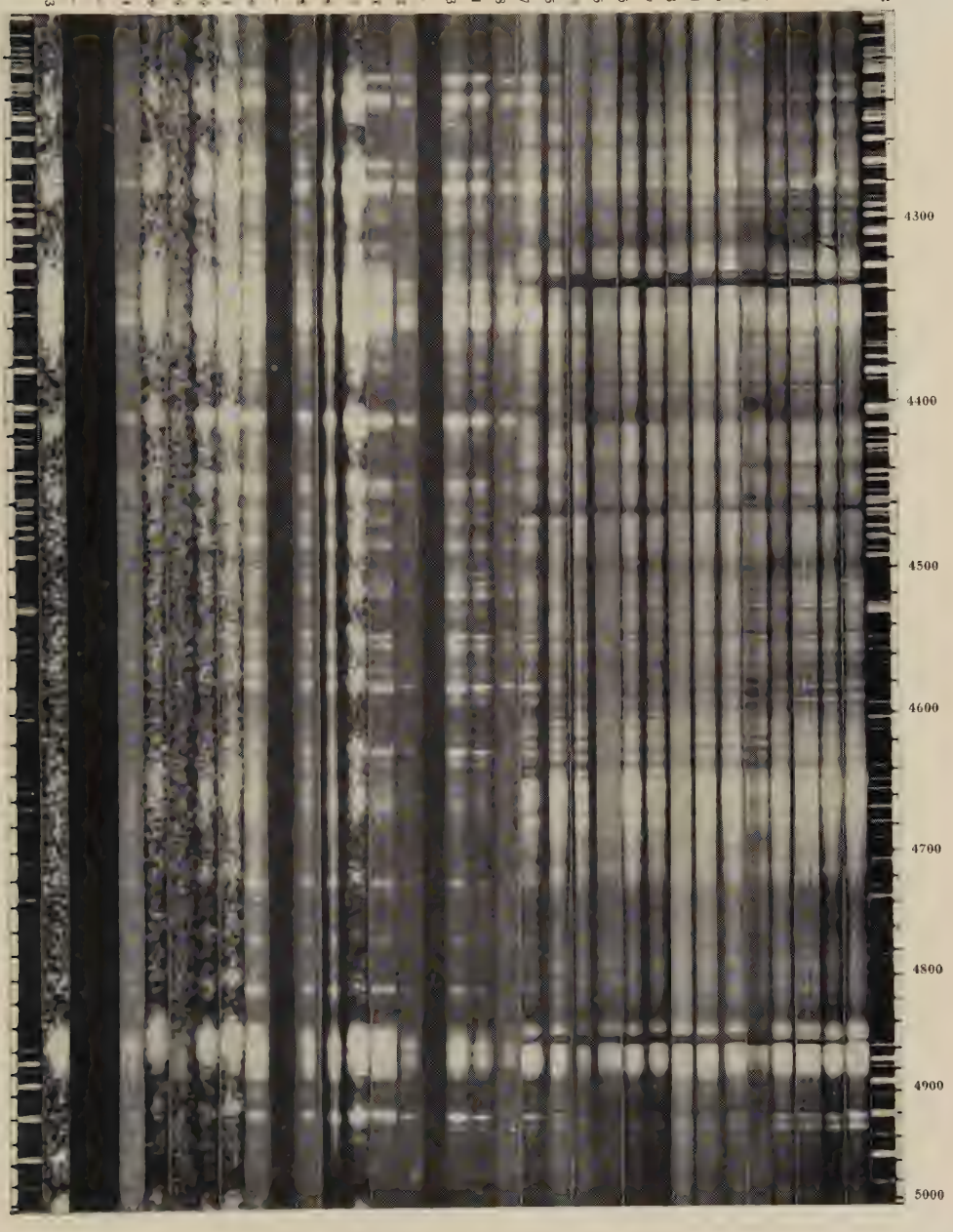
At the end of March there were present in the spectrum the following sets of lines:—

Hydrogen.	Absorptions II, IV, V, VII, XI and possibly VI.
	Emissions II, IV, XI, with double maxima in II.
Metals.	Absorptions II, VII.
	Emission II with double maxima.
B-Type Lines.	Absorptions VI, VIII.
	Emissions VI, VIII.

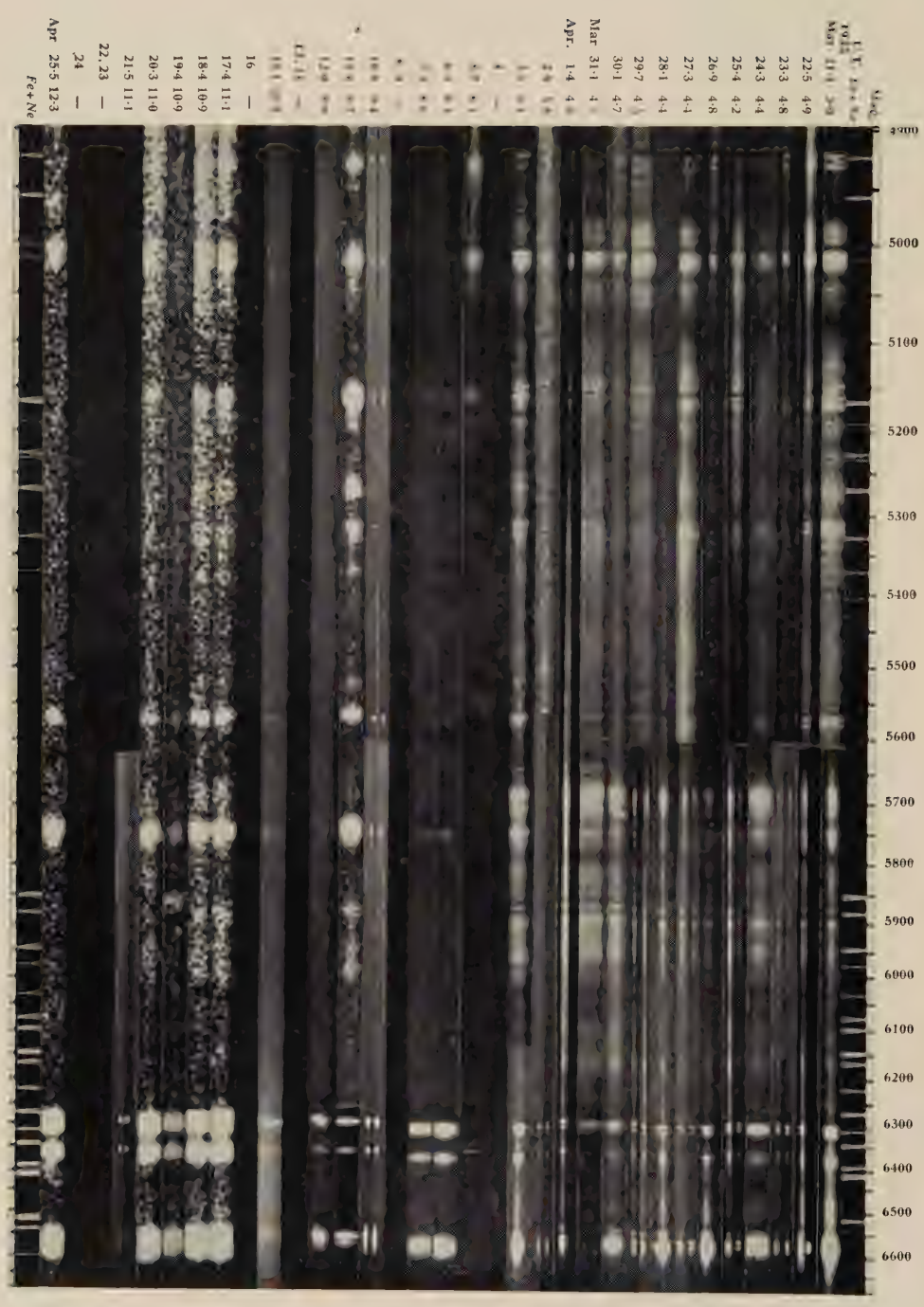
And forbidden lines of [*O*I], [*N*II], [*Fe*II] in Emission II with double maxima.

3. *The Spectral Changes as the Star Fades.*—The accompanying plates, taken from the *Atlas of Spectra of Nova Herculis* 1934‡, show the changes that accompany the beginning of the minimum phase. The continuous spectrum weakens but does not

* *Publ. Astr. Obs. Univ. Mich.*, VI, No. 12, 1937.
† Merrill, *Ap. J.*, 82, 413, 1935.
‡ Solar Physics Observatory, Cambridge, 1939.



F. J. M. Stratton The Deep Minimum in the Light-Curve of Nova Herculis 1934.



F. J. M. Stratton, The Deep Minimum in the Light-Curve of Nova Herculis 1934.

disappear (see the plate of April 12·5). The absorption lines all vanish. The Emissions VI and VIII disappear, and with them all traces of the B-type spectrum. The red maxima in Emission II weaken markedly compared with the violet maxima, and in most cases disappear. (The red maximum of 4233 A. *Fe*II does not strengthen, as would appear from the plate, but is replaced by the violet maximum of 4244 A. [*Fe*II].) The violet maxima of permitted lines move slightly to the violet to cover the positions of Absorptions II. The hydrogen Emissions II and XI survive, II being chiefly shown in the two maxima of which the violet is the stronger. The contrast between the two maxima is less marked in the case of hydrogen than in that of the metals, indicating a difference of distribution in the shells.

4. *Similar Changes in Nova Cygni* 1942.—Any explanation of the above facts must take into account the parallel sequence of events in the case of Nova Cygni 1942.* After remaining for some weeks at 9^m this nova became fainter on 1942 September 24, and in eight days dropped six magnitudes. The continuous spectrum faded, the absorption lines disappeared and, as can be seen from Plate XVII of Sanford's paper, the bright bands of *C*II, *N*II, *O*II also disappeared. There remained emission lines of *H*, *Fe*II and [*Fe*II], displaced to the violet. The hydrogen emissions extended to the red by amounts comparable with the highest observed absorption velocities to the violet. The changes closely paralleled those of Nova Herculis, and the velocities concerned −330 and −1400 km./sec. were strikingly close to those of Absorptions II and XI in the case of Nova Herculis. When the star brightened again two months later the [*Fe*II] and *Fe*II lines had faded out and been replaced by the familiar nebular lines of [*O*III]. The fading took place at the same stage of spectral development in the two stars and, as we have seen, displayed many features the same in both cases. We are driven to the conclusion that the phenomenon is not an accidental occurrence but is linked with the development of slow novæ. The sudden drop of seven magnitudes about 80 days after maximum in the case of Nova Aurigæ 1891 and the change to the typical nebular spectrum on rebrightening, strengthens the argument that we must look to the nova itself and not to some external cause for the source of the changes.

5. *Arguments for an Observing Cloud.*—The vanishing of the absorption lines and the weakening of the continuous spectrum points to an obscuring cloud passing between the Earth and the nova. The vanishing of the B-type emissions while those of the shell giving Emission II survive implies that the cloud also passes in front of the shell which gives the emissions of *C*II, *N*II, *O*II and *He*I; at the same time part at least of the shells giving Emissions II and XI remain in view from the Earth, and are also probably open to illumination from the central star.

Baade has reported an obscuring cloud with a sharp straight edge passing across Hubble's variable nebula in about four months.† In this connection it may be significant that the spectrum of the variable nebula and of its nucleus closely resembled that of Nova Aurigæ 1891 before its deep minimum ‡ with emission lines of *H*, *Fe*II, *N*II and *O*II.§ Slipher's spectroscopic evidence suggested that the nebulosity was shining by light reflected from the stellar nucleus, and Hubble‖ and Lampland have ascribed the variation occurring in different regions of the nebula to changes in its illumination from the star.¶ For the closely similar variable nebula attached to R Coronæ Australis and studied at the Royal Observatory, Helwan **, Knox Shaw has suggested the presence of an absorbing medium in motion.†† The passage of an obscuring cloud near the

* Sanford, *Ap. J.*, 97, 130, 1943.

† *Colloque International d'Astrophysique*, XIII: *Observations of Novæ*, I, p. 100, 1939 (Hermann et Cie, Paris).

‡ V. M. Slipher, *Lowell Obs. Bull.*, 81, 1918.

§ McLaughlin, *P.A.S.P.*, 53, 11, 1941.

‖ *Ap. J.*, 44, 195, 196.

¶ *P.A.S.P.*, 43, 296, 1931.

** *Bull. Helwan Obs.*, 16, 1915; 20, 1920; 30, 1924.

†† *M.N.*, 76, 647, 1916.

central star cutting off the illumination of successive regions of the nebula is indicated. Similarly the passage of a cloud formed asymmetrically between the inner and outer shells of the nova and passing across the star would meet the case of the phenomena discussed above. If the remaining emission maxima came from a part of the shell still illuminated by the star, we should expect that as they faded their velocities in the line of sight would decrease. This appears to happen: the lines of [*Fe*II] show between April 4 and 12 a drop in the velocity displacement from −455 km./sec. to −389 km./sec. Accompanying changes in brightness are discussed in a separate paper in this volume by Rutllant * and are in good agreement with this view.

6. *Inner and Outer Shells.*—Implicit in the above explanation is the view that the inner shells required for Emissions VI and VIII remain near the star and do not expand with the velocity of the particles which traverse them. This is in accordance with the conclusions reached by Tai † from a study of the contours of absorption and emission lines in Nova Herculis. He finds that Absorption and Emission IV come from a shell inside that giving Absorption II at a time when Shell IV should have overtaken and moved through Shell II, if both shells moved out with the velocities of the atoms passing through them.

From most recent novæ we have evidence that shells of matter do move out, and that the velocity of atoms streaming through them is not very different from that of the principal shell observed in the early stages of the nova's post-maximum history. But Nova Persei 1901 offers evidence that here too the outward spread of observed matter is slower than the line-of-sight velocity of the atoms in the shell at any one time. In the case of this nova there were two observed phenomena. A few months after maximum there was photographed by a number of observers a nebulosity spreading out from the star with a velocity not less than 11″ a year. In 1934 Humason ‡ obtained the spectrum of a mass of gas which was first detected by Barnard in 1916 and found to be moving out from the nova with a velocity of 0″·4 a year. He found motions in the line of sight of 1200 km./sec. Equating the velocities along and at right angles to the line of sight he deduced a parallax for the nova of 0″·00158 or a distance of over 2000 light-years. Applying this to the outward movement of the illumination observed in 1901 we get a velocity of 2×10^{11} cm./sec. or about seven times the velocity of light. This figure cannot of course be accepted, and for light spreading for six months from a body at a distance of over 2000 light-years there is no escape by assuming the illuminated nebulosity to be near enough to the Earth to give a rapid geometric spread. It follows that the velocity of outward spread of the nebulosity near Nova Persei 1901 must be less by a factor of about seven than the velocity of the gases in the line of sight.§

7. *Nature of the Obscuring Cloud.*—At the international colloquium‖ on Novæ and White Dwarfs held at Paris in 1939, the question of an obscuring cloud was discussed at some length, and Chandrasekhar suggested that ejected matter from the star might have changed its physical state and become particles or molecules. For over a month atoms of C^+, N^+, O^+, *He* and *H* had been streaming out from the nova. In view of the uncertainty as to the actual outward velocities of the shells, it is impossible to say whether these atoms would have reached the outer shell or not. They would hardly have done so if the outer shell at this stage of the star's history moved out with a velocity equal to that of the atoms forming it. In any case it seemed certain that there will be a large number of these atoms in the region between the two shells. That these atoms may have played

* *M.N.*, 105, 280, 1946.

† *M.N.*, 100, 435, 1940.

‡ *P.A.S.P.*, 46, 229, 1934.

§ [Footnote, added 1946 February 6. Since the above was written I have seen an alternative explanation of the spread of the reflected nebular illumination (P. Couderc, *Ann. d'Astrophysique*, 2, 271, 1939). This places the nebulosity in a plane crossing the line of sight to Nova Persei at an angle of 45° and at a distance of 46 light-years from the nova.]

‖ *Loc. cit.*, I, 104.

an important part in the formation of the obscuring cloud is further suggested by the case of Nova Pictoris 1925. This is the one nova, whose spectral history has been followed in detail, for which no B-type spectrum has been found bridging the change from an α Cygni to a nebular spectrum. And for this star there was no deep minimum, such as is common with slow novæ, nor periodic fluctuations in brightness typical of fast novæ in the transition stage.

In this period of oscillating brightness it has been noticed for a number of novæ that at the minima the continuous spectrum weakens, the emission bands survive and the absorption lines almost vanish to reappear at the intervening maxima. Belopolsky * noted this for Nova Persei 1901, the present writer for Nova Geminorum 1912 †, with the additional fact that at minima the bright bands covered regions of strong absorption at maxima. More recently Wyse ‡ noted the same phenomena for Nova Aquilæ 1918 and Nova Aquilæ 1927. In the case of the latter it is again a B-type spectrum whose absorption lines vanish and reappear. In fast novæ a temporary obscuration of the type discussed in this paper occurs in the transition stage, but it lasts for a few days at a time, instead of weeks or months as in the case of slow novæ. One is compelled to ask what are the differences between the physical conditions in the two cases. The spectral types in the shells are the same, so that the temperatures of the central stars are not likely to be very different.

The fast novæ are about 2^m brighter at maximum § than the slow novæ, and the transition stage commences in both cases when the star has dropped about $3^m{\cdot}5$ from its maximum brightness. So that the density of radiation is likely to be less in the case of the slow nova. The higher velocities outwards of the shells of the fast novæ || suggest a lower density in the region between the shells. It may be that the conditions between the shells in the case of slow novæ with lower intensity of radiation, slower velocities and higher density lead to the optimum density for the formation of smoke or dust particles suggested by ter Haar in his study of smoke in interstellar space.¶ The diatomic molecules involving *C*, *N* and *H* found in interstellar space, and the presence of *CN* bands in the spectrum of Nova Herculis 1934 just after its maximum brightness, support the suggestion of the preceding paragraph that the atoms of the B-type spectrum are involved in the formation of the obscuring cloud.

The presence of the continuous spectrum after the fading offers a further problem. An examination of spectrograms from a number of observatories shows it as present on April 5, 11, 12, 13, and 17 to 20. It varies in strength relative to the bright bands, being particularly strong on April 17 and very much weaker by April 25. Hubble's variable nebula already referred to similarly showed a continuous spectrum crossed by bright lines.** In this case the nebula is shining by light reflected from the stellar nucleus. It is possible that the continuous spectrum in the case of Nova Herculis after the minimum comes also from reflection from parts of the outer shell which are still receiving light from the central star. A study of the light distribution in the continuous spectrum before and after April 4 should throw light on the problem, as also a photographic and quantitative photometric study of the nucleus and nebulosity of Hubble's variable nebula and of the planetary nebulæ showing a continuous spectrum reported by Page.††

Solar Physics Observatory,
Cambridge:
1945 *October* 31.

* *Publ. Obs. cent. Nicolas*, Ser. II, 17, 107, 1903.
† *Annals of the Solar Physics Obs., Cambridge*, 3, Pt. 1, 64, 1920.
‡ *Publ. Lick Obs.*, 14, 93, 1939; 226, 1940.
§ McLaughlin, *P.A.*, 50, 10, 1942.
|| McLaughlin, *Ap. J.*, 91, 369, 1940.
¶ *Ap. J.*, 100, 288, 1944.
** *Loc. cit.*
†† *Ap. J.*, 96, 100, 1942.

SOME MEASURES OF THE SPECTRA OF NOVA HERCULIS 1934

F. Rutllant

(Received 1945 November 2)

Summary

The behaviour of the emission maxima at λλ 4929, 4919 of *Fe*II 4924 A., the emission maxima at λ 4282·1 of [*Fe*II] 4287·4 A. and the emission band at λ 4660 of *O*II and *N*II have been studied photometrically on plates taken at Cambridge between 1935 March 21 and April 5. The variations of the peak intensities of these emissions relative to the continuous spectrum and of the corresponding equivalent widths of the two lines in early April are compared with the variation of the brightness of the nova. The fading of the band at λ 4660 and the red maxima contrast strongly with the surviving violet maxima. It may further be noted that the continuous spectrum fades at first faster than the band or the red maxima.

From a number of spectrograms of Nova Herculis 1934 taken at the Solar Physics Observatory, Cambridge, a study has been made of the complex band of variable structure mainly due to *O*II and with maximum intensity near λ 4660.* The peak intensity of this emission band has been measured relative to the intensity of the continuous spectrum at λ 4740. Similar measures were made for the [*Fe*II] line at λ 4287·4 and the emissions of *Fe*II at λλ 4929, 4919 relative to the neighbouring continuous spectrum. The red components of these *Fe*II emissions weaken relative to the con-

TABLE I

Nova Herculis 1934

(Figures are relative to the neighbouring continuous spectrum, for which the value 100 is taken in each case)

No.	Date	Mag.	Peak Intensities at				Equivalent Widths of	
			λ 4919	λ 4929	Band at λ 4660	λ 4282	λ 4919	λ 4929
							A.	A.
1	1935 March 20·9	4·8	300	320	550	160	5·2	6·1
2	21·0	4·9	...	...	...	230	...	...
3	21·2	4·9	...	...	620	160	...	...
4	22·9	4·8	180	200	400	135	1·9	2·8
5	23·0	4·8	240	230	340	145	3·2	2·6
6	23·1	4·8	...	...	255	170	...	...
7	25·9	3·9	130	130	200	145	...	...
8	25·9	3·9	...	...	250	125	...	...
9	26·0	3·9	170	140	250	135	...	...
10	27·0	4·6	170	180	...	165	1·7	2·1
11	28·9	4·4	142	153	215	165	0·8	1·6
12	31·9	4·7	122	130	290	156	0·5	0·6
13	April 2·0	4·9	135	130	250	118	...	...
14	2·9	6·1	250	210	450	185	3·6	2·5
15	5·0	8·0	400	100	200	350	...	...

* *Cf.* Stratton, *M.N.*, 96, 373, 1936.

tinuous spectrum during the period between 1935 March 21 and April 2, strengthen temporarily between April 2 and 3 and then drop out, while the violet ones grow stronger. This is what would be expected if an obscuring cloud passed across the central star, still leaving part of the further side of a shell in view from the Earth. In addition to peak intensities equivalent widths have also been determined for the two maxima of *Fe*II at 4924 A. They are expressed in units of the intensity of 1 A. of the neighbouring continuous spectrum. The results are shown in Table I, which is self-explanatory.

The variations are clearly seen in the diagram of Fig. 1.

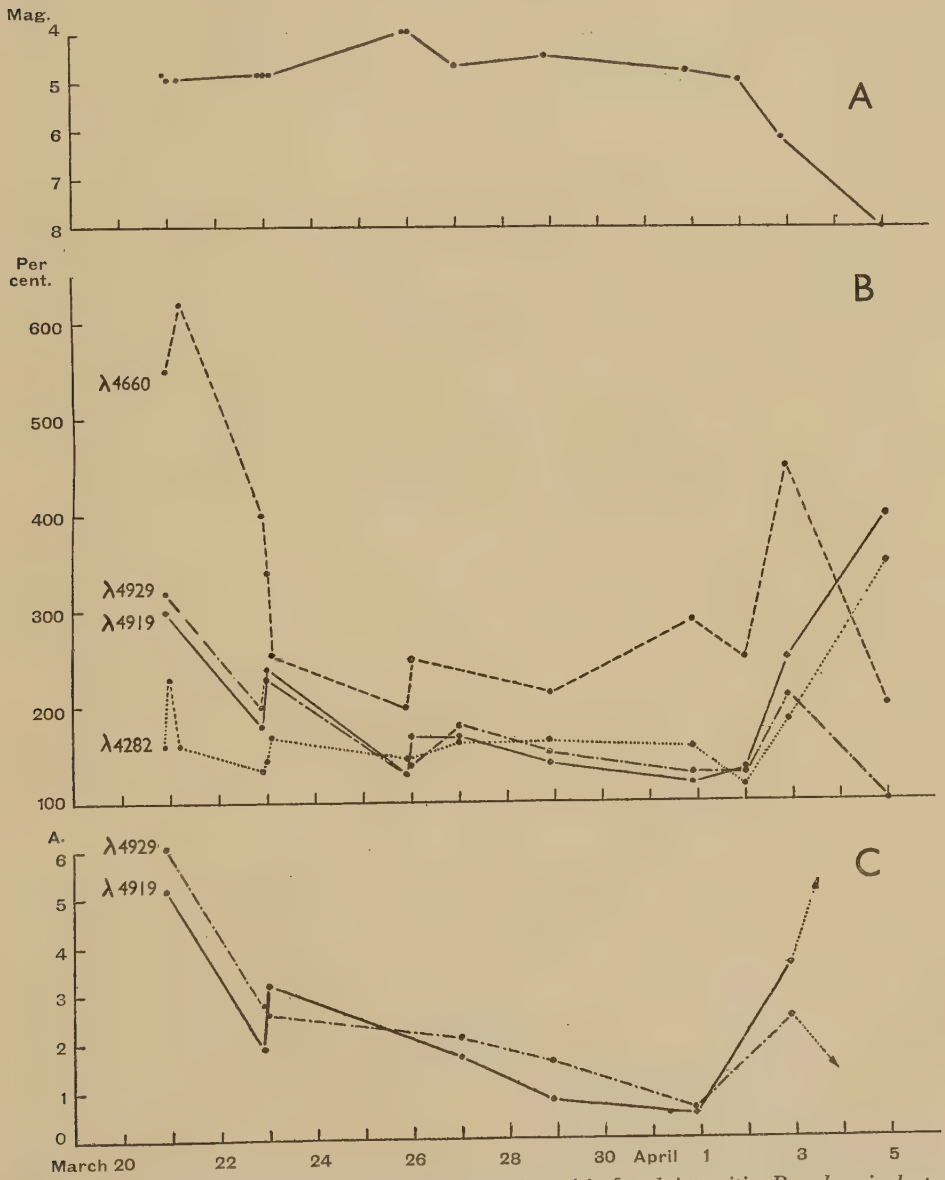


FIG. 1.—*Variations of magnitude A and line intensities [peak intensities B and equivalent widths C] of Nova Herculis* 1934 *in the period* 1935 *March* 21–*April* 5. (100 = *intensity of neighbouring continuous spectrum.*)

Solar Physics Observatory,
Cambridge:
1945 *October* 31.

INTENSITIES OF HYDROGEN *Hα* AND HELIUM D_3 IN SOLAR PROMINENCES. PAPER III

H. A. Brück and W. Moss

(Received 1945 November 2)

Summary

An attempt is made to study the effect of internal variations of excitation within prominences by tracing the variation of the intensity ratio of $I(H\alpha)/I(D_3)$ throughout a particular prominence in which intensities of *Hα* and D_3 have been measured at 123 individual points. The appearance of this prominence in the light of *Hα* and D_3 has been represented on the basis of the measured equivalent widths by two sets of isophotes which give an approximate picture of its general structure. In studying the variation of the intensity ratio of $I(H\alpha)/I(D_3)$ within the various regions of the prominence, due allowance has been made for the influence of self-absorption on *Hα*. The intensity ratio R is found to be approximately constant over one part of the prominence with a mean value $\bar{R} = 42$ which is practically identical with the value used in former calculations of the relative abundance of hydrogen and helium. In another part of the prominence where the observed isophotes show the presence of some central condensation above the chromosphere, the value of R is found to be variable. In this region existing negative density gradients are found to be more strongly pronounced in hydrogen than in helium. *Hα* appears in particular to decrease between two and three times faster with height than D_3, apparently on account of the simultaneous increase in the ionisation of hydrogen.

Photometric studies of the lines hydrogen *Hα* and helium D_3 in the spectra of prominences have been published in two previous papers.* The observed changes in the relative intensity of these two lines have been interpreted primarily as effects following from the properties of the curve of growth, and the results have been found to be consistent with the assumption of a value of about 5000° K. for the temperature of the solar boundary. Actual variations in the relative numbers of the hydrogen atoms in the 2nd and the helium atoms in the $^3P_2$ state (variations of excitation) have been taken into account in combining the results for different prominences. Signs of similar variations within one and the same prominence have been found indicated in the case of prominences where intensities of *Hα* and D_3 could be measured in a relatively large number of separate regions. Among the prominences which have been studied in Papers I and II these were, however, not sufficiently numerous to make a further examination of such internal variations of excitation possible, and the present paper is an attempt to supplement to some extent the previous work in this particular direction.

* *M.N.*, 103, 258, 1943; 105, 17, 1945. Hereafter called Papers I and II.

The paper is based on the reduction of the spectrum of a prominence of considerable extension in which intensities of *Hα* and D_3 could be measured at altogether 123 individual points compared with at best 16 points in one of the prominences of Papers I and II. The present prominence was photographed in the morning of 1944 October 2. The observational procedure was essentially the same as that which has been described in Paper I. Twenty-six cross-sections ("cuts") of the prominence were photographed in the light of *Hα* and D_3 with the two cameras of the spectroheliograph the slit of which was moved in equal steps of a quarter of a millimetre across the image of the prominence. Photographic densities in these spectra were recorded with the Cambridge microphotometer at various distances from the solar limb. The photometer tracings were reduced in the manner described previously, and values of equivalent width were determined for all measured points of the prominence.

The equivalent widths $I(H\alpha)$ and $I(D_3)$ in parts (*a*) and (*b*) of Table I are expressed in units of 0·01 A. and are relative to the intensity of the continuous spectrum of the solar centre. The individual points of the network of 123 regions in which intensities have been determined are distinguished in the table by their relative position angle in degrees, measured from one edge of the prominence, and by their height above the solar limb expressed in seconds of arc. The given values of position angle and height correspond to the known distances between the different cross-sections of the prominence and the measured positions of the photometer slit relative to the solar limb (diameter of solar image = 54 mm.).

TABLE I

Intensities of Hα and D_3 in Various Regions of a Prominence
(expressed in units of 0·01 A.)

(*a*) $I(H\alpha)$

Pos. Angle \ Height	10″	15″	20″	25″	30″	35″	40″	45″	50″
°									
0·0	3·19								
0·5	4·32	3·18	2·20						
1·0	6·10	5·42	4·15	2·52					
1·5	8·38	7·83	7·78	5·96	3·53	2·85			
2·0	11·21	10·30	10·10	8·75	6·20	4·86	2·52		
2·5	12·65	12·31	13·20	11·23	9·40	7·15	5·14	4·07	
3·0	12·88	13·11	12·84	11·73	11·66	9·71	7·47	5·54	3·15
3·5	10·78	13·09	12·81	12·13	12·11	9·75	7·68	5·97	4·10
4·0	8·32	10·89	11·47	11·09	9·70	8·49	6·76	5·48	
4·5	5·80	8·59	9·59	9·16	7·78	5·94	4·48	3·56	
5·0	4·54	6·66	8·52	8·71	7·55	5·90	3·64		
5·5	5·90	8·01	9·27	9·28	8·42	5·68	3·62		
6·0	8·11	9·51	10·60	10·12	8·15	5·48			
6·5	10·53	11·43	12·03	9·98	7·36	4·24			
7·0	9·19	9·80	10·81	8·72	6·40	3·79			
7·5	7·19	7·86	8·71	8·02	6·33				
8·0	6·73	6·71	6·12	6·36	6·03	4·06			
8·5	8·18	7·26	6·00	5·52	5·38	3·35			
9·0	8·90	7·77	5·46	4·62	3·68				
9·5	7·25	5·69	3·48						
10·0	4·82	2·41							
10·5	3·15								

(*b*) $I(D_3)$

Pos. Angle \ Height	10″	15″	20″	25″	30″	35″	40″	45″	50″
°									
0·0	0·30								
0·5	·35	0·29	0·23						
1·0	·36	·32	·27	0·27					
1·5	·33	·31	·31	·27	0·23	0·19			
2·0	·35	·33	·34	·31	·30	·25	0·16		
2·5	·36	·36	·38	·36	·35	·30	·25	0·23	
3·0	·33	·38	·39	·40	·40	·34	·30	·23	0·15
3·5	·33	·39	·41	·41	·41	·35	·28	·20	0·13
4·0	·35	·39	·40	·41	·38	·31	·21	·15	
4·5	·32	·34	·37	·37	·35	·26	·16	0·11	
5·0	·23	·26	·31	·35	·31	·20	·13		
5·5	·18	·22	·27	·32	·28	·17	0·10		
6·0	·20	·23	·25	·27	·23	·14			
6·5	·25	·27	·27	·25	·20	·11			
7·0	·25	·25	·25	·22	·17	·10			
7·5	·25	·24	·23	·24	·17				
8·0	·23	·21	·19	·20	·16	·11			
8·5	·23	·21	·18	·17	·14	0·09			
9·0	·23	·19	·15	0·11	0·08				
9·5	·19	·13	0·08						
10·0	·14	0·07							
10·5	0·11								

An attempt has been made to represent approximately the picture of the prominence as it appears in the light of *Hα* and D_3 by two sets of isophotes. These have been obtained from the data in Table I by plotting the variation of $I(H\alpha)$ and $I(D_3)$ with distance from the solar limb for all position angles represented in the table.

The result of such plots is shown in Fig. 1, in which up to five isophotes have been drawn in the case of both *Hα* and D_3 corresponding to the values of intensity which are given in the figure. Broken lines connect points whose position could safely be extrapolated from the plotted radial variations of intensities.

Both sets of isophotes show clearly though with differences the general structure of the prominence. Though it appears to be nowhere completely detached from the underlying chromosphere, the optical thickness of the prominence is obviously greater in certain central regions at higher levels than at its base, with negative intensity gradients extending in most directions from one or two central condensations.

According to the data of Table I, the intensities $I(H\alpha)$ and $I(D_3)$ vary within the prominence by a factor of about six at the most. Comparing the observed values of $I(H\alpha)$ with the curve of growth (see Papers I and II), one finds that the present prominence is in none of its measured regions bright enough for *Hα* to become appreciably affected by self-absorption. The present observations therefore do not contribute any fresh material to that aspect of the problem of line intensities in prominences which has been dealt with in our previous papers. On the basis of our earlier results, we assume the intensities of *Hα* to lie strictly on the curve of growth which corresponds to a temperature of 5000° K. as temperature of the solar boundary. Such an assumption makes it possible to determine quantitatively the amount of self-absorption which affects the measured intensities of *Hα*. One measures the deviation of the assumed curve of growth at the measured intensities from the straight line with slope one, which gives the relation between line intensities and number of emitting atoms when no self-absorption is present. In this way it is found that the maximum effect of self-absorption in the present prominence (for points near the region with position angle 3°·5 and height 15″) is about 60 per cent. of the measured intensity $I(H\alpha)$, which has to be increased by this amount to be freed from self-absorption.

Similar smaller corrections have been applied to all measured $I(H\alpha)$ in which the estimated errors of intensity measurement ($\pm$ 10 per cent.) make such corrections relevant. The values of these corrected intensities $I(H\alpha)_{corr.}$ have been compared with the intensities of D_3 in the case of which the effect of self-absorption is insignificant.

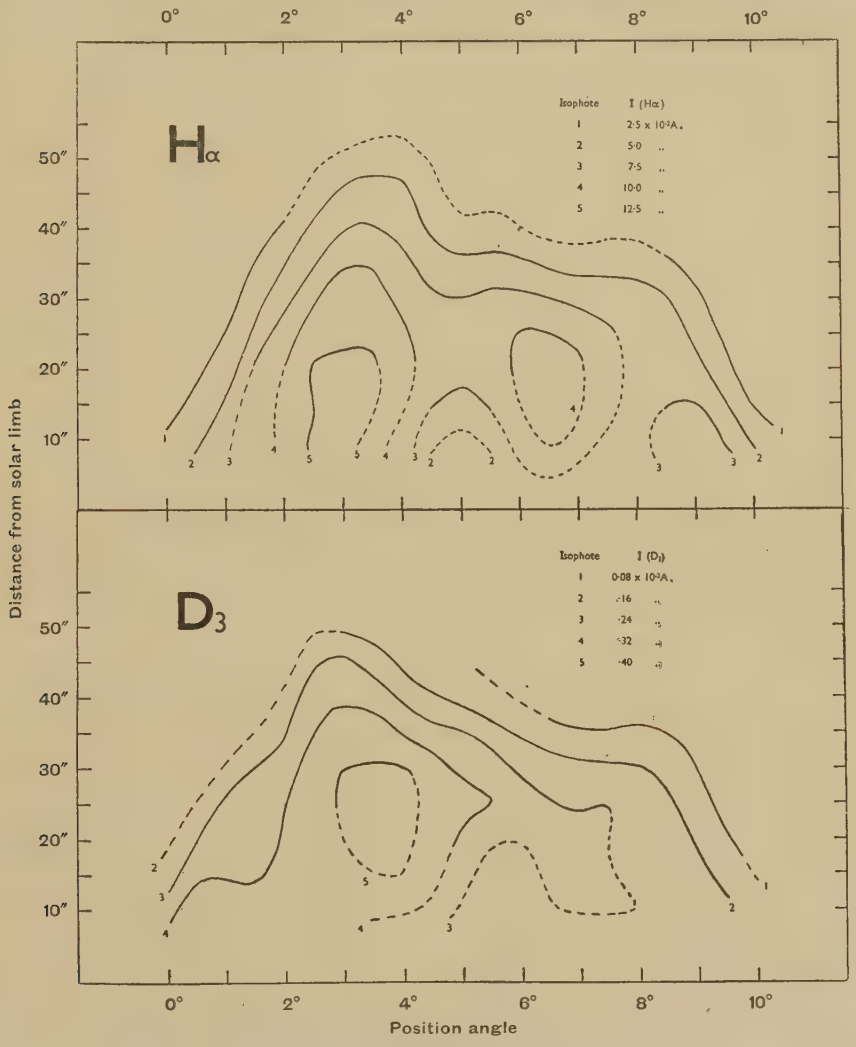


FIG. 1.—*Appearance of prominence in the light of Hα and D_3 represented by sets of isophotes. Abscissæ and ordinates are position angles and heights above the solar limb.*

Fig. 2 shows the result of such a comparison for the various regions of the prominence. The ratios $R = I(H\alpha)_{corr.}/I(D_3)$ have been plotted, and five groups of R have been distinguished corresponding to a total variation of R over a range from about 10 to 60.

Though its maximum variation is only about one-sixth of the maximum difference found in Papers I and II between the values of R for different prominences, the ratio R is seen to vary over a considerable range in certain regions of the present prominence. As due allowance has been made for the effect of self-absorption on $H\alpha$, such internal variations of R must represent actual changes in the relative numbers of those hydrogen and helium atoms which are responsible for the emission of $H\alpha$ and D_3.

Wherever the ratio R is approximately constant, that is in the region with position

angles $\geq 5°{\cdot}5$, its mean value $\bar{R}=42$ is practically identical with the value $\bar{R}=39$ which has been used by Unsöld and others in calculations of the relative abundance of hydrogen and helium in prominences. In these "normal" regions of the present prominences no definite dependence of R on height can be observed.

The case is different for the "left" part of the prominence with position angles $< 5°{\cdot}5$. In the representation of the prominence by isophotes these regions are marked

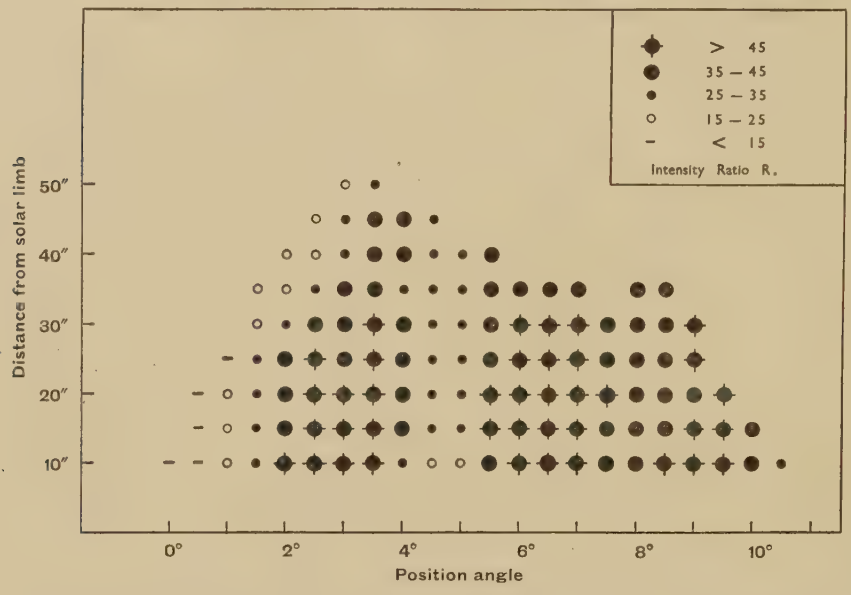


FIG. 2.—*Variations of the intensity ratio $R=I(H\alpha)_{corr.}/I(D_3)$ within the prominence. The values of $I(H\alpha)_{corr.}$ are the measured values of $I(H\alpha)$ corrected for self-absorption. Abscissæ and ordinates have the same meaning as in Fig.* 1.

by the presence of some central condensation at a level above the chromosphere, which is visible in both sets of isophotes though in these it does not appear to be in exactly the same position. In this part of the prominence, negative intensity (density) gradients observed in the case of helium are generally found to be more pronounced in the case of hydrogen. Considering in particular the existing variation of intensity with height, one finds that *Hα* decreases between two and three times faster than D_3. This result is in agreement with what had been found in the case of the prominence which among those studied previously was most suitable for the present purpose (prominence No. 20 of the list in Paper II for which intensities of *Hα* and D_3 had been measured in 13 individual points at three different levels). The explanation of the observed decrease of the ratio R with height appears to be given by the effect of the simultaneous increase in the ionisation of hydrogen.

Further determinations of line intensities are required in such prominences which can be measured at as many individual points as possible; only then can the question of excitation and its internal variation in such prominences be studied in detail for which some first quantitative measurements have been attempted in the present paper.

Solar Physics Observatory,
Cambridge:
1945 *November* 1.

ON THE STRUCTURE OF DISK-SHAPED EXTRAGALACTIC NEBULÆ I. ON THE TEMPERATURE OF INTERSTELLAR MATERIAL AND THE SHAPES OF THE NEBULÆ

F. Hoyle

(Received 1945 October 30 *)

Summary

The investigation given in the present series of papers differs markedly from former studies of the structure of extragalactic nebulæ. The main reason for this difference is that the combined mass of the stars in a nebula is here taken to be less than the total mass of the diffuse material existing between the stars (this diffuse material is referred to throughout as interstellar material). This question is discussed in detail in the Introduction to the present paper, where it is shown that the available observational evidence supports the view that the total mass of the stars forms only a small fraction of the total mass of the nebula. On this basis it is necessary to consider the physical properties and spatial distribution of the interstellar material before an attempt can be made to discuss the dynamical motions of the stars. When this has been done, the next step is to consider the conditions under which stellar condensations may form in the diffuse material. Finally, the results of these investigations may be applied to the general problems of stellar astronomy. Thus the general plan of the present series of papers is:

(1) To consider the properties of the interstellar material.
(2) To consider the condensation of stars from the interstellar material.
(3) To consider the general properties of the stars and their relation to the structure of the nebula.

The first of these three items is investigated in the present paper, the second in Paper II, and the third in Papers II, III and IV.

The properties of the interstellar material are discussed on the assumption that the material is distributed with symmetry about the axis of rotation of the nebula. The material is taken as mainly composed of hydrogen. A small proportion of heavier elements are regarded as also being present, partly in gaseous form and partly as interstellar dust. The interstellar dust, although forming only a small proportion of the mass of the material, is nevertheless physically important on account of its ability to radiate energy. It is shown that except for nebulæ of very small mass (of order 10^8 times the solar mass) the interstellar material will form a flat rotating disk with the plane of the disk perpendicular to the axis of rotation. The radius of the disk can be inferred directly from the observed distribution of stars in the nebula. The ratio of the thickness of the disk to its radius cannot be appreciably greater than 10^{-2} for such nebulæ as M33 and M31.

The following specific results are obtained in the present paper:—

(i) The relation between the thickness of the disk of interstellar material and its temperature; this is investigated in Section 2.

* Originally submitted in different form on 1945 June 18.

(ii) It is shown in Section 3 that interstellar dust is a powerful radiating agency.

(iii) The discussion of Section 4 shows that radiation by interstellar hydrogen is alone sufficient to prevent the temperature of interstellar material from exceeding 20,000° abs.

(iv) In a mixture of interstellar dust and hydrogen gas the collisions between the hydrogen and the dust transfer thermal energy to the dust and this energy is then radiated. This process is sufficiently strong to prevent the general temperature of the hydrogen from being maintained at a value of order 10,000° abs. unless some external source of thermal energy is available in addition to the absorption of radiation emitted by the stars.

(v) The result of Section 4 is applied in Section 6 to the consideration of gas pressure gradients in the plane of the disk of interstellar material. The average accelerations arising from these forces are found to be very small compared with the acceleration of the gravitational field of the nebula.

(vi) The results of Sections 2 and 4 are used in Section 7 to obtain a relation between the thickness, the diameter, and the maximum possible mass of the disk of interstellar material. From this relation it is shown that the thickness of the disk of interstellar material must be less than the thickness of the distribution of stars in the nebula, this being particularly the case in the central regions. It is then shown that the stars must have acquired large peculiar velocities, particularly in the nuclei of the nebulæ, in a direction perpendicular to the plane of the diffuse material.

(vii) An attempt to explain the origin of the peculiar velocities discussed in Section 7 is given in Section 8.

---

1. *Introduction.*—The theory of nebular structure (the word "nebula" is used throughout as equivalent to "a galaxy") developed in the present series of papers differs to a marked degree from the discussions of stellar dynamics that have hitherto been given.* It seems desirable, therefore, to explain at the outset why so different a point of view has been adopted.

The researches of Kapteyn and Schwarzschild were made at a time when the available observational data consisted almost entirely of the distribution of stellar magnitudes and motions. It is therefore not surprising that the first attempts made to interpret the dynamical properties of the stars were in terms of an assembly of discrete gravitating particles with masses of stellar order. Since this work, however, and in particular during the last ten years, much observational information has been forthcoming concerning the properties of diffuse material lying between the stars. These new results point very consistently towards the conclusion that many of the dynamical features of the stellar systems in the nebulæ are associated either wholly or in part with the effects of the diffuse interstellar material. The most important new development, in the present connection, is that the gravitational field of a nebula seems to be mainly determined by the interstellar material and not by the stars. The latter conclusion is to a large extent justified by the work of Aller, Babcock, Mayall and Wyse † on the distribution of rotational

* See, for example, W. M. Smart, *Stellar Dynamics*, Cambridge, 1938. S. Chandrasekhar, *Principles of Stellar Dynamics*, Chicago, 1942.

† H. W. Babcock, *Lick. Obs. Bull.*, 19, 41, 1939 (No. 498). N. U. Mayall and L. H. Aller, *Ap. J.*, 95, 5, 1942. A. B. Wyse and N. U. Mayall, *Ap. J.*, 95, 24, 1942. L. H. Aller, *Ap. J.*, 95, 48, 1942.

velocity and mass in M31 and M33, and also by Oort's analysis of the elliptical nebula NGC 3155.* Thus it has been shown that in these nebulæ the ratio of the mass per unit surface area of the nebular disk to the luminosity per unit area of the nebular disk increases with increasing radial distance from the centre (mass per unit surface area of the nebular disk means the mass contained in a column of unit area drawn perpendicular to the plane of the nebula). For example, when mass is measured in terms of the solar mass, and luminosity in terms of the solar luminosity, this ratio is about 10 in the nucleus of M31 and increases to a value of 76 at a distance of about 5000 parsecs from the centre.†

This result shows that either a large fraction of the mass of M31 is in the form of diffuse material (except possibly in the nucleus), or that the stars in the outer regions of the nebula are systematically smaller and less luminous than in the nucleus. There is no complete observational disproof of the latter possibility, but what evidence there is points in the opposite direction. For it is known ‡ that stars of large mass and luminosity are present in the outer parts of disk-shaped nebulæ and are absent from the nuclear regions. The observational work for M33 gives similar results to those described above for M31.

If the above remarks are applicable to the nebulæ in general, the discussion of stellar dynamics made on the basis of an assembly of discrete particles must be abandoned in favour of a theory in which the gravitational and physical properties of the interstellar material play an important rôle. At the present time, in the writer's opinion, the observational evidence, whilst not decisive, points with a fair degree of certainty towards this conclusion. Quite apart, however, from making a decision between the two theoretical points of view, it is clear that even on the least valuation of the new observational data, the evidence must be regarded as sufficient to warrant an attempt to build a theory in which diffuse interstellar material is accorded a prominent part. For these reasons the theory developed below represents a complete break with the formal development of stellar dynamics, which has increased much in complexity but not altered significantly in outlook since the time of Schwarzschild.

A further difference in point of view has been followed in discussing the formation of the spiral arms of the nebulæ. The observational work of Seyfert and Baade § is interpreted as demonstrating that the spiral arms are exceptional only in that the stars of very large luminosity are confined to these regions. This means that the arms are not regarded as containing either an exceptional quantity of material or an exceptional number of stars. Thus in the discussion given in Section 4 of Paper II it is possible to regard the distribution of diffuse material as being axially symmetric. It is seen from these remarks that the present theory is in sharp disagreement with theories that seek to explain the origin of the spiral arms in terms of rotational instability. The case against rotational instability playing any important part in affecting the large-scale distribution of material in the nebulæ is partly the observational evidence already cited, partly the discussion of Section 6 of the present paper, which shows theoretically that the gas pressure in the interstellar material is far too low for rotational instability to be important, and partly Hubble's recent demonstration that the arms "trail" behind the nucleus.‖

The third way in which the present investigation differs from previous work on stellar dynamics is that the observational results frequently used in these papers are average values for M31, M33 or the main body of the Galaxy ¶, and not for the region of the Galaxy in the neighbourhood of the Sun (that is, within about 1000 parsecs of the Sun). The argument for this procedure is that an observational value for such a quantity as

* J. H. Oort, *Ap. J.*, 91, 273, 1940.
† Wyse and Mayall, *loc. cit.*
‡ W. Baade, *Ap. J.*, 100, 147, 1944. C. K. Seyfert, *Ap. J.*, 91, 528, 1940.
§ *Loc. cit.*
‖ E. P. Hubble, *Ap. J.*, 97, 112, 1943.
¶ "Galaxy" = "our Galaxy".

the mass per unit area of the nebular disk, obtained for the neighbourhood of the Sun, cannot be regarded as a typical value that can be applied to the nebulæ in general, since the Sun probably lies in the outer fringes of the Galaxy. On the other hand, the average value of the mass per unit surface area in M31, M33 and in the main body of the Galaxy (inferred from Kepler's law applied to the period of the Sun) are found to be in such good agreement that it is natural to regard these values as typical (at any rate until further results are available for an additional number of nebulæ). This difference in outlook accounts for much of the divergence between the investigations of the present paper and Spitzer's * work on the properties of interstellar material, for in Spitzer's case the mass of interstellar dust is regarded as being comparable with the mass of interstellar hydrogen, whereas in the present discussion the interstellar dust forms only a small fraction by mass of the interstellar material—the main constituent being hydrogen.

The present work makes no claim to be anything more than a preliminary attempt to discuss the various questions that naturally arise when the point of view described above is adopted. The tentative character of the theory makes it desirable to give a broad presentation of the whole field and to avoid too much interruption of the main argument by attention to subsidiary detail. Moreover, it is important to avoid specialising the discussion beyond the stage that is justified by the observational data at present available.

2. *The Temperature of Interstellar Material.*—It will be supposed in the present paper that the rotating disk of interstellar hydrogen is axially symmetric and that the motion of the hydrogen is in a steady state. This means that the interstellar hydrogen must everywhere move in circular orbits in the nebula. The position of material in the disk will be described by the cylindrical co-ordinates r, z, ψ, where $z=0$ on the central plane of the disk and the z-axis is the axis of rotation. For an axially symmetric disk the gravitational acceleration acting on the material has a component in the z-direction and a component in the radial direction. The equilibrium of the material requires that there be other accelerations equal in magnitude to the gravitational accelerations but opposite in sign. Now the only force available to balance the gravitational force in the z-direction is a gas-pressure gradient, so that the first equation of equilibrium is

$$(\text{Gas-pressure gradient in } z\text{-direction}) = -(\text{Gravitational force in } z\text{-direction}). \quad (1)$$

The equation for equilibrium in the radial direction is

$$(\text{Gas-pressure gradient in radial direction} + \text{Centrifugal force}) = -(\text{Gravitational force in radial direction}). \quad (2)$$

It is shown in Section 6 that the gas-pressure term in (2) is very small compared with the other two terms in this equation. To a sufficient approximation equation (2) can therefore be written as

$$V^2(r, z)/r = F(r, z), \quad (3)$$

where $V(r, z)$ is the rotational velocity of the material and $F(r, z)$ is the gravitational acceleration in the radial direction. In the following work it is shown that the thickness of the disk of diffuse material is always small compared with its radius, and for this case $F(r, z)$ can be taken to a sufficient approximation as being independent of z. Thus (3) can be written as

$$V^2(r)/r = F(r). \quad (3')$$

Equation (1) can be used to obtain an important relation between the thickness of the disk and the temperature of the diffuse material. In deriving this relation the diffuse material will be regarded as composed largely of gaseous hydrogen. In addition to the hydrogen it will be supposed that the material contains a few per cent. by mass of heavy elements which are present partly as a gas mixed with the hydrogen, and partly as dust particles with dimensions of the order of 10^{-5} cm. It will be seen later as a result of the

* L. Spitzer, Jr., *Ap. J.*, 93, 369, 1941; 94, 232, 1941; 95, 329, 1942.

work of Sections 3, 4 and 5 that the temperature of the gas may differ substantially from the temperature of the dust. To distinguish between these temperatures the temperature of the gas will be denoted by T_H and that of the dust by T_D. The temperature that is significant in determining the thickness of the disk of interstellar material is T_H and not T_D. (It may be noted that if a dust particle lies outside the disk of interstellar hydrogen in the z-direction, it will eventually fall into the gaseous disk since there is no equilibrium equation of the form (1) for the dust particles. This remark, together with the supposition that the dust particles form only a small fraction by mass of the interstellar material, means that the dust cannot permanently occupy a disk with thickness greater than that of the gaseous material.)

The simplest way of presenting the following work seems to be to make a number of assumptions at the outset and to justify them at the end of the calculation.

These assumptions are:

(i) The thickness $t(r)$ of the disk of diffuse material is very small compared with the radius $r_{\max}$ of the disk.

(ii) $dt/dr << 1$.

(iii) The hydrogen is of density $\rho(r)$ and temperature $T_H(r)$, where $\rho(r)$, $T_H(r)$ are independent of z.

(iv) $\dfrac{t}{\rho(r)}\dfrac{d\rho}{dr}$ and $\dfrac{t}{T_H(r)}\dfrac{dT_H}{dr}$ are both $<< 1$.

The assumptions (ii) and (iv) mean that $t(r)$, $\rho(r)$ and $T_H(r)$ do not change appreciably in a radial displacement of order $t(r)$.

Using these assumptions, it is easy to see that the component of the gravitational force due to the disk in the z-direction is given to sufficient approximation by $-4\pi G\rho^2 z$, except near the edge of the disk (if t, ρ, T_H were independent of z, then this component of gravitational force would be given accurately by $-4\pi G\rho^2 z$, except for terms of order $t/r_{\max}$). Thus to this degree of approximation equation (1) can be written as

$$(\text{Gas-pressure gradient in } z\text{-direction}) = 4\pi\rho^2 G z, \tag{1'}$$

except near the extreme edge of the disk. This equation can be solved by using the Virial theorem, which requires the thermal energy contained in a cylinder of unit cross-section with generators in the z-direction to be equal to

$\frac{1}{2}$ [The integral of $4\pi G\rho^2 z^2$ taken between the limits $-t(r)/2$ and $t(r)/2$.]

This gives the relation

$$\frac{3kT_H\rho(r)t(r)}{2m\mu} = \tfrac{1}{2}\int_{-t(r)/2}^{t(r)/2} 4\pi G\rho^2 z^2 dz,$$

from which it is seen that

$$t(r) = \frac{9\Re T_H(r)}{\pi\mu G\rho(r)t(r)}, \tag{4}$$

where μ = mean molecular weight of the hydrogen,
m = mass of the proton,
k = Boltzmann's Constant,
G = constant of gravitation,
$\Re$ = gas constant.

Equation (4) is very important and will be used frequently throughout the whole series of papers.

Now $\rho(r)t(r)$ is the mass in grams per unit surface area of the disk, and this quantity has been determined * observationally for the nebulæ M31 and M33. (It will be noted

* H. W. Babcock, L. H. Aller, N. U. Mayall and A. B. Wyse, *loc. cit.*

that the identification of $\rho(r)t(r)$ with these measured values requires an appreciable proportion of the masses of these nebulæ to be in the form of interstellar hydrogen.) The value of $\rho(r)t(r)$ obtained from these nebulæ is about 1/5, except in the nuclear regions where a value of 1 is probably attained. An average value of about 1/5 for $\rho(r)t(r)$ is also obtained by applying Kepler's law to the motion of the Sun in the Galaxy (taking the distance of the Sun from the galactic centre as 8000 parsecs and the period as 2.10^8 years). A value of 1/5 will accordingly be regarded as a representative estimate for $\rho(r)t(r)$ except in the nuclei of the nebulæ. It is seen from (4) that an estimate for $t(r)$ can be given when this value of $\rho(r)t(r)$ is used, provided $T_H(r)/\mu$ is known. It is shown in Sections 4 and 5 that μ must be close to unity (that is, the hydrogen is mainly in the form of neutral atoms) and that T_H cannot exceed 20,000° abs. Thus it follows from (4) that $t(r)$ must be less than $3{\cdot}6.10^{20}$ cm. when $\rho(r)t(r) = 1/5$. Since the radius $r_{\max}$ will in general be of the order of, or greater than, 1000 parsecs, it is clear therefore that the assumption (i) made above must be satisfied. Furthermore, since the observational results establish * that $\rho(r)t(r)$ varies slowly with r in both M31 and M33, it can be seen from (4) that provided $T_H(r)$ also varies slowly with r then both $\rho(r)$ and $t(r)$ separately must vary slowly with r. Thus both of the assumptions (ii) and (iv) must be satisfied if $t/T_H(r).(dT_H/dr) << 1$. The remaining assumption (iii) cannot be strictly satisfied since the density must decrease continuously to zero with increasing z. This assumption can, however, be applied as an approximation for that region of the disk where the density is everywhere greater than one-half the density on the plane $z = 0$. The value of $t(r)$ given by (4) should therefore be interpreted as giving an estimate for the thickness of the inner region of the disk in which the density satisfied this condition.* An equation of the form

$$\rho(r, z) = \exp\{-q\mu G\rho(r, 0)z/RT(r)\} \tag{5}$$

may be used to estimate the variation of density with z outside this region, where q is a factor of order unity.

The above investigation shows that to determine the thickness of the disk of interstellar material it is necessary to know the distribution of the temperature T_H. In addition to the present question, the distribution of temperature turns out to be important in many other connections. A detailed discussion is therefore given in the present paper of the conditions governing the value of T_H. The method adopted below is:

(1) To consider the radiating properties of the interstellar dust alone (Section 3).
(2) To consider the radiating properties of the hydrogen gas alone (Section 4).
(3) To consider the properties of a mixture of gas and dust (Section 5).

3. It will be assumed that each dust particle can be regarded as having a definite temperature. This means that the distribution of energy among the different modes of oscillation of the atoms in the particle is in accordance with the well-known results of statistical mechanics. Furthermore, it will be supposed that the frequencies of these modes of oscillation are well distributed over the Planck spectrum corresponding to the temperature T_D of the particles, and that exchange of energy takes place between radiation and the oscillations of the material. Then the dust particle will be in equilibrium with a radiation assembly in which the energy density $I_\nu d\nu$ in the frequency range ν to $\nu + d\nu$ is given by the Planck formula

$$I_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{\exp(h\nu/kT_D) - 1}, \tag{6}$$

* The use of this approximation overestimates the thickness of the disk by a factor of order 2. This can be seen from an exact calculation using the hydrostatic equation of support, together with a special form $p \propto \rho^\gamma$ for the pressure. The presentation given above is preferred to such a calculation for the following reasons:—

(i) There is no need to introduce a special form for the pressure.
(ii) The proportionality $t(r) \propto \Re T_H(r)/\mu G\rho(r)t(r)$ is shown up more clearly.

and also there will be detailed balancing (to good approximation) in each small range of frequency.

The rate of radiation by the dust particles can be calculated by consideration of the rate of absorption of radiation that would occur if the dust particles were placed in a radiation assembly with energy density given by (6). Write $cI_\nu\kappa_\nu d\nu$ for the total radiation energy absorbed per sec. in the frequency range ν to $\nu+d\nu$ by all the dust particles lying in a column of unit cross-section with generators perpendicular to the plane of the nebula. It is seen from detailed balancing that the energy radiated by the dust particles in this range of frequency must also be $cI_\nu\kappa_\nu d\nu$. The contribution of spontaneous emission is $cI_\nu\kappa_\nu d\nu\{1-\exp(-h\nu/kT_D)\}$, the remainder of the radiated energy being due to stimulated emission. The stimulated emission for the dust particles in a nebula will be much less than in a thermodynamic assembly, but the spontaneous emission must be the same. Thus the energy radiated by the dust particles in a nebula must be estimated from the spontaneous emission, which gives

$$\frac{8\pi h}{c^2}\int_0^\infty \kappa_\nu \nu^3 \exp(-h\nu/kT_D)d\nu \tag{7}$$

for the rate of radiation by the dust particles in the unit column defined above. If $\kappa_\nu < 1$ for all values of ν giving an appreciable contribution to the integral (7), then it is seen from the definition of κ_ν that a considerable proportion of this energy must escape from the nebula. If, on the other hand, κ_ν exceeds unity in a particular frequency range ν_1 to ν_2, then the amount of energy in this frequency range that escapes from the nebula per sec. is of order

$$\frac{8\pi h}{c^2}\int_{\nu_1}^{\nu_2} \nu^3 \exp(-h\nu/kT_D)d\nu.$$

Thus the rate at which energy is radiated away from the nebula by the dust particles is

$$\frac{8\pi h}{c^2}\int_0^\infty \chi_\nu \nu^3 \exp(-h\nu/kT_D)d\nu \tag{8}$$

per unit surface area of the nebula per sec., where $\chi_\nu=\kappa_\nu$ if $\kappa_\nu<1$ and $\chi_\nu=1$ if $\kappa_\nu\geqslant 1$. In general κ_ν will be a function of the co-ordinates r, ψ depending on the distribution of the dust particles, so that in general the emission will not be uniform over the surface of the nebula.

The dust causing the absorption of radiation probably consists of metallic particles with radii of the order of, or less than, 10^{-5} cm. It has been shown * that for such particles the value of κ_ν will not increase with increasing ν faster than ν^2. Thus the expression (8) cannot be less than

$$\frac{8\pi h}{c^2}\int_0^{\nu_0} \chi'_\nu \nu^3 \exp(-h\nu/kT_D)d\nu, \tag{8'}$$

where ν_0 is a frequency such that $h\nu_0/kT_D >> 1$ and $\chi'_\nu=\kappa_{\nu_0}(\nu/\nu_0)^2$ if $\kappa_{\nu_0}(\nu/\nu_0)^2<1$ and $\chi'_\nu=1$ if $\kappa_{\nu_0}(\nu/\nu_0)^2\geqslant 1$. The results obtained below show that if ν_0 is any frequency in the photographic range the condition $h\nu_0/kT_D >> 1$ is satisfied. It is seen therefore that χ'_ν can be determined if an estimate for κ_{ν_0} is available for any given frequency ν_0 in the photographic range. Taking any such value of ν_0, the order of magnitude of κ_{ν_0} in the region of the Galaxy near the Sun can be obtained by comparing † the number of extragalactic nebulæ brighter than a given apparent photographic magnitude at the galactic pole with the number of nebulæ brighter than the same apparent magnitude at other galactic latitudes. The value of κ_{ν_0} obtained in this way is an averaged value over the whole photographic range of frequencies rather than a determination at one particular

* J. L. Greenstein, *Harvard Circular*, No. 422, 1937.

† E. P. Hubble, *The Realm of the Nebulæ*, p. 65, Oxford, 1936.

frequency. This method which gives an estimate $\kappa_{\nu_0} \simeq 1/3$ is, however, sufficiently accurate for order of magnitude calculations. It is assumed in obtaining this estimate that an appreciable proportion of galactic obscuration is due to absorption and not to scattering by the dust particles. This view is confirmed by an examination of those extragalactic disk-shaped nebulæ where the plane of the disk is approximately perpendicular to the line of sight. In many of these nebulæ dark regions are observed that cut across and sometimes obscure the normal spiral patterns, and there can be little doubt that these are regions of heavy obscuration. Now obscuration of this type could not be produced by scattering, for although sufficient scattering would make resolution into individual stars impossible it could not affect the surface brightness of the nebula over such large areas. Thus there is direct evidence that the absorption of radiation occurs to a marked degree (κ_{ν_0} of the order of, or greater than, unity) in extragalactic nebulæ, and it is therefore natural to regard an appreciable proportion of the obscuration in the Galaxy as being due to absorption.

From the above remarks it is seen that a reasonable estimate for (8′) could be obtained by putting $\kappa_{\nu_0} = 1/3$. For the present purposes, however, it is not necessary to assume a particular numerical value for κ_{ν_0}. It will be supposed, however, in the following discussion that $\kappa_{\nu_0} < 1$. The value of (8′) will then be less than the value that would arise if κ_{ν_0} were $\geqslant 1$. (All assumptions made above *reduce* the rate of radiation by the dust particles, so that the present calculations give a minimum estimate.) Then (8′) can be integrated to give

$$8\pi . 5!. \kappa_{\nu_0} h^6 T^6{}_D / c^2 \nu_0{}^2 h^5 \text{ ergs per cm.}^2 \text{ per sec.}, \tag{9}$$

since terms in $\exp(-h\nu_0/kT_D)$ can be neglected.

Let S ergs be the energy radiated by the stars per sec. per cm.^{2} of the nebular disk (that is, the energy emitted by the stars lying in a column of cross-section $A << \pi r^2{}_{\max}$ with generators perpendicular to the plane of the nebula is SA ergs per sec.). Then for $\kappa_{\nu_0} < 1$ the amount of energy absorbed by the dust particles is of order $\kappa_{\nu_0} S$ ergs per cm.^{2} per sec. If the thermal energy acquired by the dust particles in collisions with the interstellar hydrogen is neglected for the moment (it is considered below in Section 5), then an estimate for T_D can be obtained by equating $\kappa_{\nu_0} S$ to the rate of radiation (9). It is seen that since (9) gives an underestimate for the rate of radiation by the dust particles, this method of calculation will give an overestimate for T_D. Equating (9) to $\kappa_{\nu_0} S$ gives

$$T_D = \left[\frac{Sc^2 \nu_0{}^2 h^5}{960\pi k^6}\right]^{\frac{1}{6}}. \tag{10}$$

At a distance of 4000 parsecs from the centre of M31 the value of S is about 10^{-2} ergs per cm.^{2} per sec.* The value of S will vary both within any given nebula and also from one nebula to another. However, the degree of these variations is unlikely to exceed a factor of 100 by any large margin. Thus since T_D depends only on $S^{\frac{1}{6}}$, it is clear that $S = 10^{-2}$ ergs per cm.^{2} per sec. is sufficient to give a general order of magnitude estimate for T_D. Putting $\nu_0 = 7{\cdot}5 . 10^{14}$ sec.$^{-1}$, $S = 10^{-2}$ ergs per cm.^{2} per sec. in (10) gives $T_D = 53°{\cdot}9$ abs. This low value for T_D establishes the result $h\nu_0/kT_D >> 1$ used above, and shows that the dust particles are a very powerful radiating agency. This question will be raised again in Section 5.

4. Let the interstellar hydrogen satisfy a Maxwellian distribution corresponding to a kinetic temperature T_H. To a sufficient approximation we can put the energy density of radiation equal to zero. The ionization equilibrium in such an assembly differs from the results for the thermodynamic case, where there is an energy density of radiation given by replacing T_D by T_H in (6). The ionization equilibrium for the case of no radiation has been fully investigated (as also the thermodynamic case), and we may quote

* A. B. Wyse and N. U. Mayall, *loc. cit.*

the result that whilst only a very small fraction of the hydrogen will be ionized at $T_H = 10{,}000°$ abs. an appreciable proportion will be ionized at $T_H = 20{,}000°$ abs. This result is of importance in the following discussion.

The hydrogen will radiate like an ionized gas when T_H rises to 20,000° abs. The rate of radiation in this case is well known. For example, Minkowski * has given the following formula for the rate of radiation:—

$$1{\cdot}45 \cdot 10^{-27} T_H^{\frac{1}{2}}(1 + 3{\cdot}85 \cdot 10^4/T_H) n_e n_p \text{ ergs per cm.}^3 \text{ per sec.,} \qquad (11)$$

where n_e is the number of electrons per cm.^{3}, and n_p is the number of protons per cm.^{3}. The first term in this expression arises from free-free electronic transitions and the second term from free-bound transitions. When the hydrogen is almost wholly ionized,

$$n_e = n_p = \rho/m$$

for an electrically neutral assembly, and (11) becomes

$$1{\cdot}45 \cdot 10^{-27} T_H^{\frac{1}{2}}(\rho/m)^2(1 + 3{\cdot}85 \cdot 10^5/T_H) \text{ ergs per cm.}^3 \text{ per sec.} \qquad (11')$$

The rate of radiation by the hydrogen contained in a column of unit cross-section with generators perpendicular to the plane of the nebula is therefore given by

$$1{\cdot}45 \cdot 10^{-27} t T_H^{\frac{1}{2}}(\rho/m)^2(1 + 3{\cdot}85 \cdot 10^5/T_H) \text{ ergs per cm.}^2 \text{ per sec.} \qquad (12)$$

Using equation (4) of Section (2) enables (12) to be written as

$$\frac{1{\cdot}45 \cdot 10^{-27} \pi G \mu (\rho t)^3}{9 \Re m^2 T_H^{\frac{1}{2}}} \left(1 + \frac{3{\cdot}85 \cdot 10^5}{T_H}\right) \text{ ergs per cm.}^2 \text{ per sec.} \qquad (12')$$

The value of ρt for both M31 and M33 is known to be about 1/5, except near the centre where the value must be appreciably higher. Putting $T_H = 20{,}000°$ abs., $\mu = 1/2$ and $\rho t = 1/5$ in (12′) gives about 10^2 ergs per cm.^{2} per sec., which is extremely large compared with the estimate given above of 10^{-2} ergs per cm.^{2} per sec. for the rate of radiation per unit area of the nebular disk due to the stars in the nebula. Moreover, an appreciable proportion of the energy emitted in the free-free transitions and in the Balmer lines will escape from the nebula unless absorbed by the interstellar dust. It seems very unlikely, therefore, that this very high rate of radiation could be maintained, and the immediate inference is that T_H cannot be as high as 20,000° abs. unless ρt is appreciably less than 1/5. (As shown in Section 3, the dust particles are such a powerful radiating agency that even if the density of dust particles is sufficient to prevent the radiation emitted by the hydrogen escaping directly from the nebula, the radiation by the dust particles themselves will introduce exactly comparable considerations.) The degree of ionization of the hydrogen decreases so rapidly as T_H decreases below 20,000° abs. that this difficulty disappears for $T_H = 10{,}000°$ abs. The present analysis is taken in following work as showing that T_H cannot be as high as 20,000° abs.

5. The discussion of Section 3 shows that the temperature of the interstellar dust must be very small compared with 10,000° abs., provided there is no permanent supply of energy to the interstellar material *at a rate that is very large compared with the rate of radiation by the stars in the nebula.* If this latter condition is fulfilled and $T_H = 10{,}000°$ abs., then it follows that the thermal energy acquired by the dust particles in collisions with the hydrogen will almost all be radiated and lost from the nebula. (It may be noted that the work of Section 4 does not prevent T_H from being equal to 10,000° abs., although it does require T_H to be $< 20{,}000°$ abs.) The thermal energy of the hydrogen will be reduced by this process, so that in order to maintain hydrogen at a temperature of 10,000° abs. in the presence of dust particles, it is necessary to provide a supply of thermal energy

* R. Minkowski, *Ap. J.*, 96, 199, 1942.

to the hydrogen. The rate of supply of energy required to maintain such an equilibrium is calculated in the present section.

The rate at which a spherical dust particle of radius a acquires energy in collisions with the interstellar hydrogen is of order

$4\pi a^2$ [Pressure in the interstellar hydrogen] × [Mean thermal velocity of a hydrogen atom]. (13)

Thus the total energy transferred per sec. to all the dust particles in a column of unit cross-section with axis perpendicular to the plane of the nebula is given approximately by

$$4\pi\Re^{\frac{3}{2}}T_H^{\frac{3}{2}}(\rho/\mu)\Big(\sum_{a,i} a^2N(a,i)\Big), \tag{14}$$

where $N(a, i)$ is the number of dust particles of radius a and composition i in this column. A lower limit can be found for $\sum_{a,i} a^2N(a,i)$. Thus write

$$\kappa_{\nu_0} = \pi\sum_a a^2\Big[\sum_i N(a,i)Q(\nu_0, a, i)\Big], \tag{15}$$

where $Q(\nu_0, a, i)$ is defined by the condition that the energy absorbed in the range ν_0 to $\nu_0 + d\nu$ by a spherical particle of radius a and composition i from a beam of radiation of intensity E_{ν_0} is $\pi a^2Q(\nu_0, a, i)E_{\nu_0}d\nu$. Equation (15) gives an expression for the absorption coefficient defined in Section 3. Now it is known that, whatever the composition, the value of $Q(\nu_0, a, i)$ cannot be greater than a value close to 3. This value is attained for metallic particles when $a \simeq 2\pi c/\nu_0$, whilst for $a >> 2\pi c/\nu_0$ the value $Q = 1$, and for $a << 2\pi c/\nu_0$ the value of $Q << 1$. The value of Q is also $<< 1$ for dielectric particles.* Putting all the Q's equal to 3 in (15) gives the following inequality for κ_{ν_0}:—

$$\kappa_{\nu_0} < 3\pi\sum_{a,i} a^2N(a,i). \tag{16}$$

If the interstellar dust contains an appreciable fraction by mass either of dielectric particles or of metallic particles with $a << 2\pi c/\nu_0$, then the value of κ_{ν_0} will be appreciably less than the right-hand side of (16). By using (16) in (14) it follows that the rate at which thermal energy is transferred to the dust particles lying in the unit column must exceed

$$4\Re^{\frac{3}{2}}T_H^{\frac{3}{2}}\rho\kappa_{\nu_0}/3\mu \text{ ergs. per cm.}^2 \text{ per sec.},$$

which on using (4) of Section 2 becomes

$$(4\pi/27).\mu G\kappa_{\nu_0}(\rho t)^2\Re^{\frac{1}{2}}T_H^{\frac{1}{2}} \text{ ergs. per cm.}^2 \text{ per sec.} \tag{17}$$

Putting $\rho t = 1/5$, $\mu = 1$, $T_H = 10{,}000°$ abs. gives a rate of transfer of thermal energy from the hydrogen to the dust particles of about $10^{-3}\kappa_{\nu_0}$ ergs per cm.^{2} of the nebular disk per sec. This rate of transfer of energy may be compared with the total thermal energy $3\Re T_H\rho t/2$ possessed by the hydrogen in the unit column. If there were no supply of energy to the hydrogen it is clear that T_H would be appreciably reduced by the transfer of energy to the dust particles in a time of order $3.10^3\Re T_H\rho t/2\kappa_{\nu_0}$, which for $T_H = 10{,}000°$ abs. and $\rho t = 1/5$ gives a time of $2{\cdot}5.10^{11}/\kappa_{\nu_0}$ secs. This period is so short, for κ_{ν_0} of order $1/3$, that the temperature T_H will quickly fall to a value comparable with T_D unless the transfer of energy to the dust particles is compensated by a process that supplies energy to the hydrogen. Thus if T_H is maintained in thermal equilibrium at a value of order 10,000° abs. there must be a source of supply of energy to the hydrogen.

The question arises as to whether the absorption by the hydrogen of radiation emitted by the stars can be a sufficient source of supply. The most important way in which radiation can be absorbed by a hydrogen assembly is through ionisation from the ground state.† This requires quanta with energies greater than 13·5 e.V. Now unless a star

* J. L. Greenstein, *loc. cit.*

† A. S. Eddington, *I.C.S.*, p. 376, Cambridge, 1930.

is of very high surface temperature the proportion of the radiated energy in the form of quanta with energies greater than 13·5 e.V. is very small. Thus even for a typical star of class B only about 1 per cent. of the radiation emitted is in the form of quanta capable of ionizing hydrogen from the ground state, whereas for stars of smaller surface temperature the proportion of ionizing radiation is very much less than this. Thus since $S \simeq 10^{-2}$ ergs per sec. per cm.^{2} of the nebular disk gives an *average* value for the total rate of radiation by the stars, it follows that the rate of supply of ionizing radiation to the hydrogen can only be sufficient if the interstellar hydrogen under consideration happens to be in a region of the nebula containing unusually large numbers of stars of classes O and B.

The above discussion shows that when $\kappa_{\nu_0} \simeq 1/3$ and $\rho t \simeq 1/10$ the temperature T_H cannot be maintained at a steady value of 10,000° abs. unless either

(i) the interstellar hydrogen is close to a star or stars of very high surface temperature, or

(ii) some external source of supply of energy is available that is capable of giving about 10^{-3} ergs per sec. per cm.^{2} of the nebular disk. (It will be noted that this value is smaller than the value $S = 10^{-2}$ ergs per cm.^{2} per sec. used in Section 3 for the rate of radiation by the stars in the nebula. Thus this requirement will not invalidate the conclusions (1), (2) derived from Sections 3 and 4 that are given at the beginning of the present section.)

The condition $\rho t \simeq 1/10$ means that the present conclusions cannot necessarily be applied to the region of the Galaxy within about 1000 parsecs from the Sun, where the value of ρt is probably of order 10^{-2}.

It is clear that only a small proportion of the interstellar hydrogen can satisfy (i), so that if a temperature of 10,000° abs. is maintained on a large scale in the main body of a nebula where $\rho t \simeq 1/10$, then either (ii) must occur or the general obscuration must correspond to a value of κ_{ν_0} very much less than 1/3. In view of the heavy local obscuration that occurs in many extragalactic nebulæ the latter alternative does not appear very plausible. It is therefore of interest to consider the possibility (ii). Two suggestions can be made:

(*a*) Collisions occur between the main nebular disk and appendages of the nebula (such as diffuse material in globular clusters). These subsidiary bodies are to be regarded as lying outside the nebular disk except during brief periods when for dynamical reasons they pass through the disk.

(*b*) Encounters occur between the nebula and diffuse material lying between the nebulæ.

The possibility (*a*) is discussed in Sections 7 and 8 and the possibility (*b*) is investigated in detail in Paper IV. It may be noted here that the process (*a*) must occur very generally since globular clusters in particular seem to be frequently associated with the nebulæ. Moreover, as shown in the Introduction to Paper IV, there is some observational support for (*b*).

It is necessary to examine in relation to the above discussion the available observational evidence for values of T_H as high as 10,000° abs. The temperature of the interstellar material in the neighbourhood of the Sun (that is, at distances from the Sun that are small compared with the distance of the galactic centre) has been shown by two different observational methods to be of order 10,000° abs. in certain regions. Thus Dunham has estimated the temperature from the broadening of interstellar absorption lines *, whilst Struve and Elvey † have observed hydrogen emission lines in many regions of the Milky Way. Both of these investigations show that T_H has a value of order 10,000° abs. in the regions in question. Unfortunately these results cannot be regarded as an indica-

* T. Dunham, *Proc. Amer. Phil. Soc.*, **8**, 277, 1939.

† O. Struve and C. T. Elvey, *Ap. J.*, **88**, 364, 1938; **89**, 119 and 517, 1939.

tion of the value of T_H for the nebulæ in general, since, as pointed out above, ρt is probably smaller than 10^{-1} in the solar neighbourhood. Thus conditions are much more favourable for a high value of T_H in the neighbourhood of the Sun than is the case, for example, in the main body of M31 and M33 (this can be seen from the dependence of (17) on ρt). Further information can be obtained from the obscuration observed in nebulæ where the line of sight lies approximately in the plane of the nebular disk (spindle-shaped nebulæ). The observed bands of obscuration suggest that the disk of interstellar dust is at least 100 parsecs thick in certain cases. Now since the dust cannot permanently occupy a disk of greater thickness than that of the interstellar hydrogen, these observations suggest a thickness of the order of 100 parsecs for the disk of hydrogen. This argument indicates that the interstellar hydrogen attains a temperature of order 10,000° abs. in certain cases. (It would be possible to overcome the present argument by supposing that the disk of interstellar material is curved, for in this case the apparent thickness of the bands of obscuration may be due to the curvature and not to the thickness of the disk. Thus the argument is far from being conclusive.) It is not necessary for the whole of the hydrogen to be at this temperature, since the bands of obscuration could evidently be produced if T_H were of order 10,000° abs. in a sufficient number of localised regions.

If the tentative conclusions reached above are taken together with the requirement discussed in Section 2 of Paper II that the value of T_H must be very small in regions where stars are condensing (of order 10° abs.), it is seen that either there must be large variations of temperature with time, or there must be large fluctuations from one region to another within the nebulæ. The process (*a*) given above would be capable of producing both intermittent variations of temperature and also fluctuations between different regions in a nebula. The process (*b*) could evidently produce variations of temperature with time.

It remains to prove a result that has been assumed in the above discussion: that dust particles will not evaporate when T_H rises to a value of order 10,000° abs. This can be seen immediately, since the rate of transfer of energy from the hydrogen to the dust particles given by the above work is less than the value $S = 10^{-2}$ ergs per cm.^{2} per sec. for the rate of radiation per cm.^{2} of the nebular disk by the stars in the nebula. But it was shown in Section 3 that the temperature of the dust particles must be less than 100° abs. even if the dust absorbs the whole of the radiation from the stars. It follows *a fortiori* that the supply of energy to the dust particles arising from collisions with the interstellar hydrogen cannot raise the temperature of the dust particles above 100° abs., which is very small compared with the melting-point of the particles.

6. Rotational instability has frequently been invoked to explain such properties as the spiral arms of the nebulæ. It is well known that the instability must arise from the combined effect of both the centrifugal forces and the gas-pressure gradient. The force due to the gas-pressure gradient is important, because an appreciable proportion of the energy required for the ejection of material must be derived from the thermal energy of the material. To examine this question in the case of a nebula, we compare the maximum *average* value for the gas-pressure gradient parallel to the plane of the nebula with the acceleration due to the gravitational field of the nebula. For rotational instability to be important these two accelerations must be of comparable magnitude.

The condition of Section 4 that the temperature of the interstellar hydrogen must be less than 20,000° abs. places an important restriction on the average acceleration due to the gas-pressure gradient, which is of order $\Re T_H / r_{mb}$, where r_{mb} is the radius of the main body of the nebula. Putting $r_{mb} = 2000$ parsecs and $T_H = 10{,}000°$ abs. gives a maximum average acceleration of order 10^{-10} c.g.s. units. This value may be compared with the acceleration of order 10^{-7} c.g.s. units arising from the gravitational field of a typical nebula. This comparison shows that it is very improbable that the large-scale distribution of material in a nebula could be altered by rotational instability. In contrast with this situation the acceleration due to the gas pressure in a rotationally unstable star is of the same order as the gravitational acceleration.

The above remarks apply to the acceleration averaged over a large region of the nebula. It is of course possible that the acceleration due to gas pressure in a localised region may exceed the average value by a considerable factor, and that this might produce some localised changes in the distribution of material. But in considering large-scale changes in the distribution of material the average acceleration must evidently be used.

7. It was shown in Section 4 that 20,000° abs. may be taken as a general upper limit for the temperature T_H of the interstellar hydrogen. From equation (4) of Section 2 it can be seen that $T_H < 20{,}000°$ abs. requires

$$\bar{t} < 25/\overline{\rho t} \text{ parsecs}, \tag{18}$$

where $\bar{t}$ and $\overline{\rho t}$ are averaged values for the nebula. If M is the mass of the nebula, then $M \simeq \pi r^2_{mb} \overline{\rho t}$, where r_{mb} cm. is the radius of the main body of the nebula, in which ρt is comparable with $\overline{\rho t}$ (outside the main body ρt falls rapidly with increasing distance from the centre). If r_{mb} is measured in parsecs instead of centimetres, the corresponding equation for M is

$$M \simeq \pi . 10^{37} r^2_{mb} \overline{\rho t}.$$

Thus from the inequality (18) for $\bar{t}$ it follows immediately that

$$M < 2{\cdot}5\pi . 10^{38} r_{mb} (r_{mb}/\bar{t}). \tag{19}$$

The quantity $\bar{t}/2r_{mb}$ is a measure of the degree of flattening of the disk of interstellar hydrogen, and the inequality (19) requires the mass of a disk of hydrogen of given radius and a given degree of flattening to be less than a known numerical upper limit. The values of this upper limit for a set of values of r_{mb} and $\bar{t}/2r_{mb}$ are given in the following table:—

TABLE I

r_{mb} (parsecs)	$\bar{t}/2r_{mb}$	Upper Limit to the Mass of Hydrogen / Mass of the Sun
400	1/4	$\pi . 10^8$
700	1/6	$2{\cdot}6\pi . 10^8$
2000	1/6	$7{\cdot}5\pi . 10^8$
2000	1/10	$1{\cdot}25\pi . 10^9$
6000	1/10	$3{\cdot}75\pi . 10^9$

Since the main contribution to the mass of a nebula is taken to be due to the interstellar hydrogen, it follows that an upper limit to the mass of a nebula of given radius and degree of flattening can be obtained from the present considerations. This argument may be applied in an inverted form. Thus, given the mass and radius of a nebula, the inequality (19) can be used to give an upper limit to the value of $\bar{t}/2r_{mb}$. This upper limit to the degree of flattening of the disk of interstellar hydrogen is given in the following table for M33, M31 and the Galaxy:—

TABLE II

	M	r_{mb}	Upper Limit to $\bar{t}/2r_{mb}$
M33	2.10^{43} gm.	1,000 parsecs	$1{\cdot}96 . 10^{-2}$
M31	2.10^{44} gm.	6,000 parsecs	$1{\cdot}18 . 10^{-2}$
Galaxy	4.10^{44} gm.	8,000 parsecs	$7{\cdot}85 . 10^{-3}$

The upper limits to $\bar{t}/2r_{mb}$ given in Table II are less by a factor of order 10 than the observed degrees of flattening *of the stellar distributions* in these nebulæ (which are comparable with the values of $\bar{t}/2r_{mb}$ given in Table I). The immediate inference is that the observed thickness of the distribution of stars must be greater than the thickness of the disk of interstellar hydrogen. It might be argued that this conclusion could be avoided if a large fraction of the masses of the nebulæ were in the form of stars. There are several important reasons, however, why this suggestion is unsatisfactory:

(1) The observational evidence available does not support this view.

(2) The assumption that a large fraction of the mass of a nebula is in the form of stars does not really avoid the conclusion reached above, since the above discussion must in any case be valid at the time when the condensation of the stars took place.

A further consideration that supports the conclusion arrived at from Tables I and II arises from equation (4) of Section 2, which shows that t is inversely proportional to ρt (t is the thickness *of the disk of hydrogen* throughout this discussion). Thus t will be unusually small in regions of a nebula where ρt is unusually large. The observational data suggests that * ρt is unusually large near the centre of the nebula, so that a disk of hydrogen will be thinnest at the centre. This is in direct opposition to the observed shapes of the stellar distributions, which are much thicker at the centre than in the main body of the nebulæ. This argument, together with the results given in Tables I and II, suggests that the distribution of stars changes with time, and in particular that the thickness of the stellar distribution is markedly thicker than that of the interstellar material in which the stars condense. This means that the stars must acquire peculiar velocities in a direction perpendicular to the plane of the nebular disk. The peculiar velocities necessary to explain the observed stellar distributions are in some cases very large. Thus if t_s is the thickness of the distribution of stars (the subscript s is used for the thickness of the stellar distribution to distinguish from t, which is the thickness of the hydrogen disk), it can be shown that the peculiar velocities must be of order $\sqrt{(2\pi G\rho t t_s)}$. This expression follows immediately from considering the motion of the star in a direction perpendicular to the plane of the nebula, it being supposed that $t_s << r_{mb}$, and the gravitational force of the nebula in this direction being taken as $2\pi G\rho t$. Near the centre of M31 the value of ρt is about 1 and t_s is about 1000 parsecs, whilst at a distance of 5000 parsecs from the centre $\rho t \simeq 1/10$ and $t_s \simeq 250$ parsecs. Thus the peculiar velocity required to explain the thickness of the stellar distribution at the centre of M31 is about 350 km. per sec., and the value required at a distance of 5000 parsecs from the centre is about 550 km. per sec. These peculiar velocities are so large that the following questions evidently require an answer:—

(*a*) What is the origin of these large peculiar velocities?

(*b*) Why are the peculiar velocities acquired by stars near the centre of a nebula much larger than the peculiar velocities of the stars in the outer regions?

A tentative attempt to answer these questions is given in the following section.

8. It was shown in Section 7 that the thickness of the disk of interstellar material must be less than the observed thickness of the stellar distributions in disk-shaped nebulæ. Near the centres of the nebulæ in particular the thickness of the distribution of stars is very much greater than the thickness of the disk of interstellar material. It appears, therefore, that the stars must have acquired large peculiar velocities in a direction perpendicular to the plane of the nebula, and that these velocities in some cases must exceed 100 km. per sec. The present remarks apply explicitly to the disk-shaped nebulæ, but it seems clear that a similar situation also arises for the elliptical nebulæ. The argument for these nebulæ is that at the time of condensation of the stars the temperature of the diffuse material must have been small (see Section 2 of Paper II). Thus at this time both the diffuse material and the stars must have occupied a thin disk. A subsequent increase in the temperature of the diffuse material would produce a general expansion of the disk, but unless the mass of the nebula is less than $5.10^8 \odot$ this process is severely limited by the condition that T_H must not exceed 20,000° abs. It appears, therefore, that unless the masses of nebulæ of small ellipticity are very small the stars in these nebulæ must have acquired large peculiar velocities; the situation in these cases being very similar to the

* A. B. Wyse and N. U. Mayall, *loc. cit.*

problem of the peculiar velocities in the central regions of the massive disk-shaped nebulæ.

The large peculiar velocities described above present such a serious problem that it was at one time suggested that the radiation emitted by the elliptical nebulæ could not be of stellar origin. This method of avoiding the difficulty is no longer possible, for Baade * has recently succeeded in resolving both the central regions of M31 and a number of elliptical nebulæ. The origin of the large peculiar velocities that must occur in these nebulæ is a matter of outstanding importance, and it is clearly desirable to compile a list of all suggestions that seem to offer hope of overcoming the present difficulty.

It will be assumed that the masses of the globular clusters are of order $10^8\odot$, so that the velocity of escape from the surface of a cluster is of the order of, or greater than, 100 km. per sec. This assumption is supported by the fact that the richest globular clusters have luminosities that are almost comparable with the faintest elliptical nebulæ. Further support for this assumption can be obtained by considering the passage of a globular cluster through the nebular disk (for dynamical reasons the clusters must frequently pass through the plane of the nebula). It is clear that the clusters would quickly be scattered by these collisions unless their mean densities were much higher than the mean density in the nebula. If the clusters contain an appreciable fraction by mass of diffuse material, they will punch a hole in the nebula at each collision. The mass acquired by the cluster in such a collision will be of order $\pi D^2\rho t/4$, where D is the diameter of the cluster. The time between successive collisions can be shown to be of order $2\sqrt{(s/\pi G\rho t)}$, where s is the "radius" of the orbit of the cluster. Thus the mass acquired by the cluster in 10^9 years due to the present process is of order

$$3.10^{16}D^2G^{\frac{1}{2}}(\pi\rho t)^{\frac{3}{2}}/8s^{\frac{1}{2}}.$$

Putting $\rho t=\frac{1}{2}$ for the central regions of a nebula, $s=2000$ parsecs and $D=30$ parsecs gives a mass of about $10^8\odot$. Thus quite apart from the initial mass possessed by a globular cluster it seems probable that a cluster will pick up a mass of the required order in a period of 10^9 years.

It is clear that in addition to sweeping up a quantity of material at each collision with the nebular disk the gravitational field of the cluster will also have a powerful scattering effect on nearby stars. Thus the stars lying within an area of at least πD^2 of the nebular disk will acquire peculiar velocities of order 100 km. per sec. at each collision of the globular cluster with the nebular disk. These peculiar velocities will in general have components in a direction perpendicular to the plane of the nebula, and are of the order of magnitude required to explain the observed thicknesses of the distribution of stars in the nebulæ. A single globular cluster will have about 50 collisions with the nebular disk in 10^9 years, so that in this period the stars over an area of about $50\pi D^2$ of the nebular disk will acquire the required peculiar velocities. (Owing to the rotation of the nebula the cluster will not affect the same stars at each collision.) Thus the number of globular clusters that a nebula of radius 1000 parsecs must possess to ensure that in 10^9 years an appreciable fraction of the stars within the nebula acquire peculiar velocities of order 100 km. per sec. is about 20 (taking $D=30$ parsecs for each cluster). The numbers of clusters required is of course less if a period greater than 10^9 years is allowed for the process. A similar result applies to the nuclear regions of massive disk-shaped nebulæ such as M31 and the Galaxy.

The marked concentration of globular clusters towards the centre of M31 supports the considerations given above, since it enables the great thickness of the stellar distribution in the nucleus to be satisfactorily explained. Indeed central concentration of the globular clusters provides a possibility of explaining the lenticular shapes of the nebulæ of types "later" than E7. Moreover, it may be possible to explain the shapes of elliptical nebulæ of types "earlier" than E7 by supposing that the globular clusters are not markedly

* W. Baade, *Ap. J.*, 100, 137, 1944.

concentrated towards the central regions in these nebulæ (a reasonable supposition on account of the small dimensions of these nebulæ). Observations of the globular clusters associated with elliptical nebulæ are necessary to confirm this suggestion. It seems clear that the repeated collisions between a nebula and its associated globular clusters will have the effect of gradually reducing the dimensions of the orbits of the clusters and will consequently tend to increase their central concentration.

An immediate inference from the above discussion is that accretion of interstellar material must be negligible for stars in the nuclei of the nebulæ on account of their very high peculiar velocities. It is natural to suggest, therefore, that the observed absence of highly luminous stars from the nuclei may be due to the fact that accretion must be negligible (see Introduction to Paper IV).

St John's College,
Cambridge:
1945 *June* 15.

II. ON THE CONDENSATION OF STARS, THE LUMINOSITY FUNCTION, AND THE DISTRIBUTION OF BRIGHT STARS

Summary

The general plan stated in the *Summary* to Paper I is continued in the present paper. That is, having discussed the properties of the interstellar material in Paper I, we now proceed to consider the properties of stars in relation to the interstellar material. The first step is to discuss the condensation of stars from the interstellar material and then to investigate the distribution of the stars both with respect to mass and to their position in the nebula.

The detailed results established below are:

In Section 2 the problem of the condensation of stars is discussed in relation to considerations of temperature and angular momentum for the case where the disk of interstellar material is in equilibrium. It is shown that the temperature of the interstellar material must be very low (of order 100° abs.) at the time when stars are condensing. The angular momentum of a stellar condensation turns out to be so large that a rotationally stable star cannot be formed unless

(*a*) The star is formed in a special region of the nebula where the differential rotation of the interstellar material is unusually small.

(*b*) Accretion of interstellar material occurs on a sufficient scale before the onset of rotational instability.

The results of Section 2 are applied in Section 3 to a discussion of the luminosity function.

The process (*a*) together with the arguments of Section 3 are used in Section 4 to explain the origin of the bright ring of stars that occurs in the outer regions of many nebulæ. A general account of the distribution of the highly luminous stars is also given in Section 4 with special reference to the origin of the spiral arms.

---

1. *Introduction.*—The chief assumptions underlying the work of the present paper are:—

(1) That the main contribution to the mass of a nebula is due to the interstellar hydrogen.

(2) That the distribution of the interstellar hydrogen is approximately axially symmetric in all nebulæ except those of irregular form.

The second of these two hypotheses immediately raises the question of whether the bright stars with absolute magnitude of order -5 can be distributed in a spiral pattern in such nebulæ. For this to be the case it is necessary for the formation of the spiral arms to be intimately connected with the origin of the bright stars. In order to deal with this problem it is important to consider first the general distribution of the stellar population with respect to mass. Thus it is known from a statistical study of stars in the Galaxy * that if $N(M)dM$ represents the number of stars with masses lying between M and $M+dM$, then $N(M)$ decreases rapidly with increasing M. This statement is valid down to values of M of about $M_\odot/5$ (below $M_\odot/5$ the faintness of the stars makes it difficult to obtain adequate statistical data. It is probable that $N(M)$ continues to increase as M decreases below $M_\odot/5$.) The adoption of the short time scale of about 10^{10} years for the age of the universe means that a star of mass $< M_\odot$ cannot have exhausted by nuclear transformations any large fraction of the hydrogen that was originally present in the star at the time of its condensation from diffuse gaseous material. Thus it would seem necessary to interpret the form of the function $N(M)$ as a property of the condensation process. If a condensation process can be obtained that favours the formation of stars of small mass, then the origin of the bright stars of large mass can be explained either by the occurrence of exceptional conditions for condensation or by large accretion of interstellar hydrogen taking place after condensation to a compact stellar body has taken place.

The present paper is concerned with a discussion of the problems described above. In Section 2 the condensation of stars is considered, and the results of Section 2 are applied in Section 3 to show that in general the masses of the stars at the time of condensation must be small. The origin of the bright stars of large mass is discussed in Section 4 and the relation with the spiral arms is discussed. Although the latter problem is not solved it is possible to account for many of the observed forms of spiral structure without introducing specialised hypotheses. In particular it can be shown why the arms "trail" behind the nucleus, thereby avoiding the difficulties of theories based on rotational instability.

Before proceeding to a discussion of these questions it may be noted that there is important observational support for the assumption (2) on which the present work is based. This supporting data may be summarised in the following three points:—

(*a*) It has been shown by Seyfert † that the light from the spiral nebulæ can be divided into two parts. Thus whereas the light in the visible and ultraviolet regions of the spectrum follows the spiral pattern there is a general background of red light, arising presumably from stars of small mass and luminosity, that follows an approximately axially symmetric distribution. It follows from this that the small stars do not show the spiral structure.

(*b*) On account of the rapid increase of stellar luminosity with mass as given by the mass-luminosity relation, it is dangerous to infer the general distribution of mass in a nebula from the distribution of massive stars of large luminosity. Thus, for example, the small stars mentioned above in (*a*) may well have a greater total mass than the combined mass of the comparatively few stars of large luminosity. The extreme character of an extrapolation from the distribution of the bright stars can also be seen from the remark that if only one part in 10^5 of the mass of M31 were aggregated into stars of mass $10\ M_\odot$ ($M_\odot$ is the solar mass) the combined luminosity of these stars would be comparable with the observed total luminosity of M31.

(*c*) Massive stars of large luminosity are absent from the nuclei of the nebulæ, although the value of ρt is known to have its largest value in the nuclei.

* "Galaxy" = "our Galaxy". Note also that in the present paper M is used to denote a mass of stellar order, whereas in Paper I M was used for the mass of the nebula.

† C. K. Seyfert, *Ap. J.*, 100, 147, 1944.

2. *The Condensation of Stars.*—Let r, ψ, z be cylindrical co-ordinates describing the position of points in the nebula, where the z-axis is perpendicular to the plane of the nebula with $z = 0$ on the central plane. It was shown in Paper I that the thickness of the disk of interstellar hydrogen must be small compared with radius r_{mb} of the main body of the nebula. Thus both the radial gravitational acceleration $F(r)$ and the rotational velocity $V(r)$ can be taken as independent of z. Moreover, since the nebula is axially symmetric $F(r)$ and $V(r)$ are also independent of ψ and satisfy equation (3′) of Paper I.

The discussion given below differs from the work of Jeans.* In his work on the formation of stars Jeans considers the stability of a uniform gas with a view to determining whether or not variations of density will develop within the gas. If variations of density are found to arise in regions of appropriate dimensions it is considered that stars must necessarily form. In the present work, on the other hand, it is supposed that fluctuations of density are always present in the gas, and the conditions the angular momentum and gas pressure must satisfy in order that stars may be formed are investigated.

Let the density of the interstellar material in a small spherical region of radius a, with centre at distance r from the centre of the nebula, be larger than the density in surrounding regions. The radius a is to be regarded as less than t and very much less than r_{mb}. Let interest be fixed in this particular sample of the interstellar material, and let this material contract under its own gravitational field (the conditions under which this may occur are examined below). As the contraction proceeds the total angular momentum of the material about its centre must remain constant if no new material is added to the condensation during its contraction (the external gravitational field of the nebula acting on a spherical distribution of material cannot generate angular momentum about the centre of the sphere). In the present section it will be supposed that no new material is added to the condensation during the condensation. (The effect of adding new material is considered in detail in the Appendix to the present paper.) Then angular momentum is conserved during the contraction, and the average angular momentum per unit mass about the centre must remain equal to its initial value at the time of formation of the condensation. This initial value of the angular momentum per unit mass about the centre of the sphere has evidently a magnitude of order $a^2 \mid dV/dr \mid$. It follows therefore that if u is the equatorial velocity of rotation of the condensation at any stage during the contraction then u is given by

$$u \simeq (a^2/R) \mid dV/dr \mid, \tag{1}$$

where R is the radius of the condensation at the stage in question of the contraction. If the mass of the material in the condensation is of stellar order and is written as M then

$$M \simeq 4\pi a^3 \rho/3, \tag{2}$$

where it is supposed that the density of the material in the spherical region before any contraction takes place is comparable with, but greater than, the general density ρ of the interstellar material in surrounding regions. Eliminating a from (1) and (2) gives

$$u \simeq \left(\frac{3M}{4\pi\rho}\right)^{\frac{2}{3}} \frac{1}{R} \left|\frac{dV}{dr}\right|. \tag{3}$$

The average value of $\mid dV/dr \mid$ for both M31 and M33 is about 2.10^{-15} sec.$^{-1}$. If this value is accepted as typical then (3) can be written in the form

$$u \simeq 2{\cdot}8 \,.\, 10^{-3} \rho^{-\frac{2}{3}} \left(\frac{M}{M_\odot}\right)^{\frac{2}{3}} \frac{R_\odot}{R} f(V), \tag{4}$$

where $R_\odot$ is the solar radius and $f(V) = \mid (dV/dr)/\overline{(dV/dr)} \mid$ is the magnitude of the ratio of the differential rotational velocity in the nebula at distance r from the centre of the nebula to the average value of dV/dr for the whole nebula.

* J. H. Jeans, *Astronomy and Cosmogony*, p. 337, Cambridge, 1936.

The next step in calculating the velocity u is to determine the value of ρ in regions where stars are condensing. An estimate for ρ can be obtained by considering the assumption made above that gas pressure gradients do not prevent the contraction of the stellar condensation. The average value of the gas pressure in the condensation before contraction takes place will be of order $\Re\rho T_H/\mu$ (it being evidently reasonable to suppose that the gas pressure in the condensation is of the same order as the pressure in the surrounding material). Now it would clearly be unplausible to suppose that during the contraction the gas pressure remained exactly constant throughout the condensation, particularly as even in the early stages of the contraction it is to be expected that the central density will be greater than the average density in the condensation. Thus it is reasonable to regard the pressure as fluctuating by a factor of at least 2 between the centre and the surface of the condensation. It appears therefore that a gas pressure gradient of order $\Re\rho T_H/\mu a$ will occur within the condensation. Now if the gravitational field of the material in the condensation is to play an important part in the concentration of the material, the gravitational acceleration which is of order GM/a^2 must be comparable with, or greater than, the acceleration of order $\Re T_H/\mu a$ arising from the gas pressure gradient. This will be the case if

$$\Re T_H/\mu < GM/a. \tag{5}$$

Eliminating a by means of (2) gives

$$T_H < 1{\cdot}1\mu GM^{\frac{1}{3}}(\rho t)^{\frac{1}{3}}/\Re, \tag{6}$$

where equation (4) of Paper I is also used in deriving (6). Thus when $\rho t = 1/5$, $\mu = 1$ the following inequality for T_H arises from the requirement that gravitational forces exceed the gas pressure gradients,

$$T_H < 17{\cdot}5(M/M_\odot)^{\frac{1}{3}} \text{ in degrees absolute.} \tag{7}$$

It appears therefore that it is reasonable to take $T_H < 100°$ abs. over the whole range of stellar masses (taking $T_H < 100°$ abs. when $(M/M_\odot)^{\frac{1}{3}}$ is of order unity would seem to make ample allowance for errors that may arise through taking the gas pressure in the condensation as $\Re T_H\rho/\mu$ and the gas pressure gradient as $\Re\rho T_H/\mu a$).

If the condition $T_H = 100°$ abs. is maintained throughout the thickness of the nebula then from equation (4) of Paper I it is seen that $\rho = 1{\cdot}1 \,.\, 10^{-19}$ gr. per cm.^{3} when $\rho t = 1/5$. Putting this value for ρ in the above equations gives the following equations for u and a:

$$u \simeq 1{\cdot}2 \,.\, 10^9 \left(\frac{M}{M_\odot}\right)^{\frac{2}{3}} \frac{R_\odot}{R} f(V) \text{ cm. per sec.} \tag{8}$$

$$a \simeq 1{\cdot}7 \,.\, 10^{17}(M/M_\odot)^{\frac{1}{3}} \text{ cm.} \tag{9}$$

Now for main series stars the radius of a star is approximately proportional to the two-thirds power of its mass, so that if R in (8) is taken as the final radius of the star after contraction has proceeded until a stellar equilibrium state of the main series type is reached, then $(M/M_\odot)^{\frac{2}{3}}R_\odot/R$ must be of order unity. Thus if $f(V)$ is also of order unity it is seen that u is of order 10^9 cm. per sec., which is far higher than the velocity at which a main series star would become rotationally unstable.

It might be thought at first sight that the very high values of u given by (8) could be reduced by any required factor simply by taking the temperature of the interstellar material to be sufficiently low, thereby increasing the value of ρ in (4) to the required extent. This, however, is not the case, since the thickness t of the nebula is inversely proportional to ρ, whereas the value of a given by (2) is proportional to $\rho^{-\frac{2}{3}}$. Thus as ρ increases the ratio a/t increases until the value unity is reached. When this stage is reached no further reduction of u is possible by continuing to increase ρ. The value $\rho = 1{\cdot}1 \,.\, 10^{-19}$ gr. per cm.^{3} used in the above calculation together with $\rho t = 1/5$ gives $t = 2{\cdot}2 \,.\, 10^{18}$ cm., which may be compared with the value of a given by (9). It is seen

that in the above calculation a is already of order $t/10$ and it is therefore to be expected that increasing ρ further could only reduce u below (8) by a factor of about 10 at most. This conclusion can be confirmed by direct calculation. Thus if $a \geqslant t$ the material of the stellar condensation may be regarded as occupying a disk-shaped portion of the nebula of radius a and thickness t, and equation (2) has to be replaced by

$$M \simeq \pi a^2 \rho t, \tag{10}$$

Equation (1) for u remains valid, so that the expression for u when a is eliminated from (1) and (10) is

$$u \simeq \frac{M}{\pi \rho t R} \left| \frac{dV}{dr} \right| \tag{11}$$

and this can be expressed immediately in the form

$$u \simeq 9{\cdot}1 . 10^7 \frac{M}{M_\odot} \frac{R_\odot}{R} f(V) \text{ cm. per sec.}, \tag{12}$$

where $\rho t = 1/5$ and $\overline{|dV/dr|}$ is again taken as 2.10^{-15} sec.$^{-1}$. The value given by (12), which clearly represents a minimum estimate for u, is still greater than the critical value for rotational instability when R is taken as the radius of a main series star of mass M and $f(V)$ is of order unity.

The above discussion shows that it is not possible for rotationally stable stars to condense in a nebula in equilibrium unless

(i) $f(V)$ is appreciably less than unity.

(ii) A large amount of material with negligible angular momentum about the centre of the stellar condensation is accreted before the onset of rotational instability —that is, before the stellar condensation has contracted to such a degree that rotational instability occurs.

The possibility (i) is discussed below in Section 4 and the possibility (ii) is investigated in Section 3.

The further question arises in relation to these conclusions as to whether the difficulty of rotational instability can be overcome when we allow new material to be added to the condensation during contraction. To avoid interrupting the main argument this question is examined in an Appendix, where it is shown that such a process, if it occurs at all, can only arise in very rare cases.

3. *The Luminosity Function.*—3.1. One of the most interesting problems of astronomy concerns the form of the luminosity function. It was pointed out in the Introduction to the present paper that the number of stars $N(M)dM$ with masses lying between M and $M+dM$ decreases rapidly with increasing M for $M > M_\odot/5$, and that the adoption of the short time scale makes it very probable that this dependence on M represents a property of stellar condensation. It is therefore of importance to consider in what way the process of stellar condensation can determine the distribution of stellar masses. Before considering this question in detail it may be noted that the often-quoted remark due to Eddington that radiation pressure may impose an upper limit on the masses of the stars does not provide an answer to the present problem. For it is known from the work of Cowling * on the stability of stars, that when the mean molecular weight μ of the stellar material is less than unity this limit, if it exists at all, can only be operative for masses greater than $100\, M_\odot$. Thus since it is to be expected that the stars at the time of condensation will be largely composed of hydrogen, giving a value of μ close to $1/2$, it is clear that radiation pressure must be irrelevant in the present connection. The problem is not to explain why stars of mass $1000\, M_\odot$ do not occur, but why, for example, stars of mass $2M_\odot$ should be much less frequent than stars of mass $M_\odot/2$.

There are two ways of attacking the present problem. It may be assumed that

* T. G. Cowling, *M.N.*, 96, 42, 1935.

irregularities of density in the interstellar material will in general be of such a distribution of sizes that the stellar condensations arising from these irregularities possess the required distribution with respect to mass, or alternatively it may be argued that although irregularities of density occur over a wide range of dimensions some process intervenes to prevent irregularities of too large dimensions from condensing into stars. The difficulty with the former suggestion is that since the mass depends on the cube of the linear dimensions of the irregularity it would be necessary for the dimensions of the irregularities to have an extremely peculiar distribution. For example, if a is the radius of a spherical irregularity giving a stellar condensation of mass $M_\odot/5$, then a spherical irregularity of radius $3a$ occurring in material of the same density will give a steller condensation of mass $5{\cdot}4\ M_\odot$. Accordingly an explanation of the form of the function $N(M)$ based on the distribution of the dimensions of irregularities of density in the interstellar material could only be satisfactory if a cogent reason could be advanced to show why the probability of occurrence of a condensation of radius a must be far higher than the probability of occurrence of a condensation of radius $3a$. In view of the variations of both temperature and density that are likely to occur within a nebula it is considered improbable that an adequate solution of the present problem can be obtained along these lines.

An attempt will be made below to show that there are important factors which in general prevent the formation of stable stars when the mass of a stellar condensation is appreciably greater than $M_\odot$. It will be supposed that the case of $f(V)$ appreciably less than unity is of comparatively rare occurrence. Thus although there is no restriction on the mass of a star in this special case it is supposed that only a small proportion of stars are formed by this process. Apart from this small group of stars all other stars will be regarded as satisfying equation (8), so that rotationally stable stars cannot be formed unless appreciable accretion of interstellar material, with negligible angular momentum about the star, occurs before the star contracts sufficiently for rotational instability to arise. This suggestion seems hopeful, for it is shown below that the time of contraction to the rotationally unstable state depends approximately on the inverse fourth power of M, and that the time of contraction is very short for stellar condensations of mass $5\ M_\odot$ but is very long for stars of mass $M_\odot/5$.

Before discussing this question it is of interest to consider what is likely to happen to a star that becomes rotationally unstable. What appears to be an excellent example of the break up of a rotationally unstable star has recently been investigated in detail by Baade and Minkowski.* In this particular case it is possible to observe directly the nebulous material that was thrown off by the supernova of 1054. The mass of this material is found to be of order $15\ M_\odot$, so that the star must have thrown off an appreciable proportion of its original mass. The observations are consistent with the material being showered out in all directions lying approximately within a plane (the observations are not consistent with isotropic emission of material since the observed shape of the nebulosity is not spherical). The cause of rotational instability in the case of this star seems to be that the supply of hydrogen was exhausted by nuclear transformations and the star was contracting towards a white dwarf state. As contraction proceeded the ratio of centrifugal force to gravitational force increased until the two forces became comparable in magnitude. At this stage a process similar to that described by Kuiper for the β Lyræ system seems to have occurred in which material spills out of the system of closed equipotential surfaces into regions where the equipotentials are open and extend to infinity.† The suggestion that rotational instability occurred during the contraction of the star from a main series state to a white dwarf state is directly confirmed by the estimates given by Minkowski (*loc. cit.*) for the radius and the density of material in the remaining nucleus. Rotational instability occurring through the collapse of stars that have exhausted their supply of hydrogen must be a frequent source of novæ and supernovæ.

* W. Baade, *Ap. J.*, 96, 188, 1942. R. Minkowski, *Ap. J.*, 96, 199, 1942.

† G. P. Kuiper, *Ap. J.*, 93, 133, 1941.

where B is independent of M and R (this neglects the slight dependence of the coefficient of opacity on M and R). A numerical estimate for B can then be obtained by making the following assumption: That a star of mass $M_\odot$ will have a luminosity of $L_\odot\mu^{7\cdot5}/(1-X^2)$ when R has contracted to $R_\odot$. The reasonableness of this assumption is shown by putting $\mu=1$, which gives the requirement that a contracting star of solar mass and similar molecular weight to the Sun will have the solar luminosity when $R=R_\odot$ (the factor $1-X^2\simeq 1$ when $\mu=1$). When the present assumption is satisfied, it can be shown by an argument similar to that leading to (20) that B is given by

$$B=2GM_\odot^2(1-X^2)/R_\odot L_\odot\mu^{7\cdot5}. \qquad (22)$$

It is seen from (22) that B has the same dependence on X, μ as the numerical factor in (20). Numerical estimates for τ can be obtained from (21) and (22) when M and R are given. This will now be considered.

It is known * that when energy generation is by the carbon-nitrogen cycle there is a homology relation

$$\frac{\text{Radius of a main series star of mass } M}{R_\odot}\simeq\left(\frac{M}{M_\odot}\right)^{\frac{3}{4}}. \qquad (23)$$

Thus if R in (21) is put equal to the radius of a main series star of mass M in which energy generation is *by the carbon-nitrogen cycle* then

$$[\text{Time of contraction to the main series state}]\simeq\left[\frac{2GM_\odot^2(1-X^2)}{R_\odot L_\odot\mu^{7\cdot5}}\right]\left(\frac{M_\odot}{M}\right)^4. \qquad (24)$$

Equation (24) will give an overestimate for the time of contraction of a stellar condensation to a state of rotational instability since, for the condensations discussed in Section 2, rotational instability must occur before the main series state is reached. The numerical factor in (24) can be simplified by noting that X will in general lie between 0·6 and 0·95 and for values of X in this range $\mu^{7\cdot5}/(1-X^2)$ is about 1/10. Thus we obtain the following overestimate for the time of contraction to a state of rotational instability:—

$$5.10^{16}(M_\odot/M)^4 \text{ sec.} \qquad (25)$$

Since energy generation must be by the carbon-nitrogen cycle when $M=2M_\odot$, the time of contraction to a state of rotational instability for this mass must be less than 10^7 years. This time is too short for the large amount of accretion to occur that would be necessary to prevent rotational instability occurring. It may therefore be concluded that a stellar condensation of mass $2M_\odot$ will contract until rotational instability occurs (except in the special case where $f(V)$ is appreciably less than unity). This situation will be even more marked for stars of mass greater than $2M_\odot$ since the time of contraction (25) decreases rapidly with increasing M.

For $M\leqslant M_\odot/2$ it is no longer permissible to apply the homology relation (23) since for such values of M the energy generation will be mainly by the direct synthesis of helium from protons and not by the carbon-nitrogen cycle. The radius of a main series star in which energy is generated by direct synthesis of helium from protons is approximately independent of M. It follows therefore that (25) has to be replaced by

$$10^{16}(M/M_\odot)^{3\cdot5} \text{ sec.} \qquad (26)$$

when $M\leqslant M_\odot/2$. Thus when $M=M_\odot/5$ the time of contraction to the main series state is about 10^{11} years, which is longer than the age of the universe on the short time scale. It seems probable therefore that, quite apart from accretion slowing down the rate of rotation, the contraction process in small stars takes so long that the state of rotational instability may never be reached.

* See, for example, F. Hoyle and R. A. Lyttleton, *M.N.*, **102**, 177, 1942.

The above discussion shows that

(*a*) Condensations of mass greater than $2M_\odot$ will reach a state of rotational instability in time less than 10^7 years. This time is too short for accretion of interstellar hydrogen to prevent rotational instability occurring. It follows that stars of mass greater than $2M_\odot$ cannot be formed by direct condensation from the interstellar material.

(*b*) Condensations of mass $M_\odot/5$ require a time comparable with the age of the universe to contract to a main series state. This provides ample time for accretion to reduce the rate of rotation of the condensation. Moreover, even if accretion does not occur, the time of contraction is so long that there must be many condensations of small mass in a contracting state at any given time. These results provide an adequate explanation of the luminosity function since a stable star will be formed only if its initial mass is $< 2M_\odot$.

The important assumptions underlying this statement are:

(1) That only a *small* proportion of stars condense in regions where $f(V)$ is appreciably less than unity.

(2) There is a second assumption implicit in the above remarks, that only a small proportion of stellar condensations of small mass, $M < M_\odot$, subsequently grow into stars of large mass, $M \simeq 10M_\odot$, by accretion of interstellar material.

It is to be emphasised that although the great majority of stars are of small mass, the small proportion of stars of large mass may give the main contribution to the total luminosity of a nebula. This is due to the strong dependence of the luminosity of a star on its mass as given by the mass-luminosity relation.

4. *The Distribution of Bright Stars in the Nebulæ.*—4.1. The main contribution to the luminosity of a nebula in the visible and ultraviolet parts of the spectrum comes from a small group of stars with masses greater than $2M_\odot$ (except possibly in the nuclei of the nebulæ where the density of stars of small mass must be very high). Thus normal photographs of the nebulæ give the distribution of a small group of exceptional stars, which is not necessarily related to the general distribution of material within the nebulæ. As Baade has recently shown *, it is only when the infra-red part of the spectrum is used that it is possible to obtain an idea of the distribution of the far more numerous stars of small luminosity. Even if there were no supporting evidence it would be plausible to suggest that stars possessing masses far above the average may also have a special spatial distribution. The importance of considering the factors determining the distribution of bright stars may be judged from the fact mentioned in the Introduction, that if one part in 10^5 of the mass of M31 were aggregated into stars of mass $10M_\odot$ the combined luminosity of these stars would be comparable with the total brightness of M31.

According to the discussion given in Sections 2 and 3 of the present paper a star must satisfy one of the two following requirements if it is to be exceptionally massive and luminous:—

(*a*) It condenses in a region in which $f(V)$ is appreciably less than unity, and is exceptionally massive at the time of condensation.

(*b*) The star, although not unusually massive at the time of condensation, becomes massive through accretion of interstellar hydrogen.

The case of $f(V)$ appreciably less than unity has an interesting application. The condition $dV/dr = 0$ which requires $rF'/F = -1$ will be satisfied at certain specific values of r. ($F(r)$ is the radial component of the gravitational acceleration of the nebula.) Thus since $F(r)$, $V(r)$ are independent of ψ in an axially symmetric nebula, it follows that a

* W. Baade, *Ap. J.*, 100, 137, 1945.

where B is independent of M and R (this neglects the slight dependence of the coefficient of opacity on M and R). A numerical estimate for B can then be obtained by making the following assumption: That a star of mass $M_\odot$ will have a luminosity of $L_\odot\mu^{7\cdot5}/(1-X^2)$ when R has contracted to $R_\odot$. The reasonableness of this assumption is shown by putting $\mu=1$, which gives the requirement that a contracting star of solar mass and similar molecular weight to the Sun will have the solar luminosity when $R=R_\odot$ (the factor $1-X^2\simeq 1$ when $\mu=1$). When the present assumption is satisfied, it can be shown by an argument similar to that leading to (20) that B is given by

$$B=2GM_\odot^2(1-X^2)/R_\odot L_\odot\mu^{7\cdot5}. \tag{22}$$

It is seen from (22) that B has the same dependence on X, μ as the numerical factor in (20). Numerical estimates for τ can be obtained from (21) and (22) when M and R are given. This will now be considered.

It is known * that when energy generation is by the carbon-nitrogen cycle there is a homology relation

$$\frac{\text{Radius of a main series star of mass } M}{R_\odot}\simeq\left(\frac{M}{M_\odot}\right)^{\frac{3}{4}}. \tag{23}$$

Thus if R in (21) is put equal to the radius of a main series star of mass M in which energy generation is *by the carbon-nitrogen cycle* then

$$[\text{Time of contraction to the main series state}]\simeq\left[\frac{2GM_\odot^2(1-X^2)}{R_\odot L_\odot\mu^{7\cdot5}}\right]\left(\frac{M_\odot}{M}\right)^4. \tag{24}$$

Equation (24) will give an overestimate for the time of contraction of a stellar condensation to a state of rotational instability since, for the condensations discussed in Section 2, rotational instability must occur before the main series state is reached. The numerical factor in (24) can be simplified by noting that X will in general lie between 0·6 and 0·95 and for values of X in this range $\mu^{7\cdot5}/(1-X^2)$ is about 1/10. Thus we obtain the following overestimate for the time of contraction to a state of rotational instability:—

$$5.10^{16}(M_\odot/M)^4 \text{ sec.} \tag{25}$$

Since energy generation must be by the carbon-nitrogen cycle when $M=2M_\odot$, the time of contraction to a state of rotational instability for this mass must be less than 10^7 years. This time is too short for the large amount of accretion to occur that would be necessary to prevent rotational instability occurring. It may therefore be concluded that a stellar condensation of mass $2M_\odot$ will contract until rotational instability occurs (except in the special case where $f(V)$ is appreciably less than unity). This situation will be even more marked for stars of mass greater than $2M_\odot$ since the time of contraction (25) decreases rapidly with increasing M.

For $M\leqslant M_\odot/2$ it is no longer permissible to apply the homology relation (23) since for such values of M the energy generation will be mainly by the direct synthesis of helium from protons and not by the carbon-nitrogen cycle. The radius of a main series star in which energy is generated by direct synthesis of helium from protons is approximately independent of M. It follows therefore that (25) has to be replaced by

$$10^{16}(M/M_\odot)^{3\cdot5} \text{ sec.} \tag{26}$$

when $M\leqslant M_\odot/2$. Thus when $M=M_\odot/5$ the time of contraction to the main series state is about 10^{11} years, which is longer than the age of the universe on the short time scale. It seems probable therefore that, quite apart from accretion slowing down the rate of rotation, the contraction process in small stars takes so long that the state of rotational instability may never be reached.

* See, for example, F. Hoyle and R. A. Lyttleton, *M.N.*, **102**, 177, 1942.

The above discussion shows that

(*a*) Condensations of mass greater than $2M_\odot$ will reach a state of rotational instability in time less than 10^7 years. This time is too short for accretion of interstellar hydrogen to prevent rotational instability occurring. It follows that stars of mass greater than $2M_\odot$ cannot be formed by direct condensation from the interstellar material.

(*b*) Condensations of mass $M_\odot/5$ require a time comparable with the age of the universe to contract to a main series state. This provides ample time for accretion to reduce the rate of rotation of the condensation. Moreover, even if accretion does not occur, the time of contraction is so long that there must be many condensations of small mass in a contracting state at any given time. These results provide an adequate explanation of the luminosity function since a stable star will be formed only if its initial mass is $< 2M_\odot$.

The important assumptions underlying this statement are:

(1) That only a *small* proportion of stars condense in regions where $f(V)$ is appreciably less than unity.

(2) There is a second assumption implicit in the above remarks, that only a small proportion of stellar condensations of small mass, $M < M_\odot$, subsequently grow into stars of large mass, $M \simeq 10M_\odot$, by accretion of interstellar material.

It is to be emphasised that although the great majority of stars are of small mass, the small proportion of stars of large mass may give the main contribution to the total luminosity of a nebula. This is due to the strong dependence of the luminosity of a star on its mass as given by the mass-luminosity relation.

4. *The Distribution of Bright Stars in the Nebulæ.*—4.1. The main contribution to the luminosity of a nebula in the visible and ultraviolet parts of the spectrum comes from a small group of stars with masses greater than $2M_\odot$ (except possibly in the nuclei of the nebulæ where the density of stars of small mass must be very high). Thus normal photographs of the nebulæ give the distribution of a small group of exceptional stars, which is not necessarily related to the general distribution of material within the nebulæ. As Baade has recently shown *, it is only when the infra-red part of the spectrum is used that it is possible to obtain an idea of the distribution of the far more numerous stars of small luminosity. Even if there were no supporting evidence it would be plausible to suggest that stars possessing masses far above the average may also have a special spatial distribution. The importance of considering the factors determining the distribution of bright stars may be judged from the fact mentioned in the Introduction, that if one part in 10^5 of the mass of M31 were aggregated into stars of mass $10M_\odot$ the combined luminosity of these stars would be comparable with the total brightness of M31.

According to the discussion given in Sections 2 and 3 of the present paper a star must satisfy one of the two following requirements if it is to be exceptionally massive and luminous:—

(*a*) It condenses in a region in which $f(V)$ is appreciably less than unity, and is exceptionally massive at the time of condensation.

(*b*) The star, although not unusually massive at the time of condensation, becomes massive through accretion of interstellar hydrogen.

The case of $f(V)$ appreciably less than unity has an interesting application. The condition $dV/dr = 0$ which requires $rF'/F = -1$ will be satisfied at certain specific values of r. ($F(r)$ is the radial component of the gravitational acceleration of the nebula.) Thus since $F(r)$, $V(r)$ are independent of ψ in an axially symmetric nebula, it follows that a

* W. Baade, *Ap. J.*, 100, 137, 1945.

circular ring of bright stars will be formed at each of these values of r. Now in any symmetric nebula there must always be at least one value of r at which $dV/dr = 0$. This arises since near the centre dV/dr must be > 0, whereas in the extreme outer fringes of the nebula dV/dr must be < 0. The measured rotational velocity as a function of r for M31 shows that * there are three values of r at which $dV/dr = 0$, two of these values lie within 1000 parsecs of the centre and the third is at more than 6000 parsecs from the centre. In the case of M33 there is one value of r at a distance of about 1000 parsecs from the centre where $dV/dr = 0$. From these remarks it is seen that there must always be at least one circular ring of bright stars in an approximately axially symmetric nebula (there is a proviso attached to this statement that is given in the following paragraph). This ring of bright stars will usually occur in the outer regions of a nebula since rF'/F will normally be close to unity in the main body of a nebula. The present result is in satisfactory agreement with observation, for the occurrence of a bright outer ring of stars at some distance outside the nucleus is one of the most persistent features associated with the extra-galactic nebulæ.†

There is an important condition attached to the above argument, since it is assumed that the density of the interstellar material in the outer regions of a nebula where $rF'/F = -1$ is sufficient to permit the formation of stars of large mass. If the density in this region were very much less than the mean density for the nebula, it is clear that the above conclusions might have to be modified. An analysis of the mass distribution for M33 shows how this may occur ‡, for it is found that the rotational velocity curve for this nebula can be explained by a density distribution that falls off rapidly with increasing r in just those regions where $f(V)$ is appreciably less than unity. In such cases the ring structure may be faint, broken, or absent.

4.2. The arguments of 4.1 suggest that it may be possible to obtain a theory of the spiral arms of the nebulæ without supposing that the distribution of material in the nebulæ differs markedly from a state of axial symmetry and without assuming the orbits of the stars in the arms to differ appreciably from circles. Thus if it can be shown that the processes (*a*) or (*b*) of 4.1 occur only in the spiral arms, or alternatively that massive stars outside the arms do not undergo sufficient accretion to avoid exhausting their supply of hydrogen by thermo-nuclear reactions, it is clear that an adequate explanation of the spiral arms can be given. In the former case massive stars occur only in the arms, whereas in the latter case although massive stars may condense in regions lying outside the arms their lifetimes must be very short.

The present suggestion seems attractive because recent observational evidence is in serious contradiction with other theories based on the idea of rotational instability. Thus quite apart from the theoretical objection raised in Section 6 of Paper I, Hubble has shown § that the arms "trail" in *all* nebulæ (16) that are suitably orientated for an

* H. W. Babcock, *Lick Obs. Bull.*, **19**, 41 (No. 498), 1939.

† The approximation of taking the angular momentum per unit mass as of order $a^2 dV/dr$ in a stellar condensation is sufficiently accurate in the main body of a nebula. In the outer regions a further term arising from the Coriolis force has to be taken into account (the Coriolis force being due to the rotation in the nebula of the centre of gravity of the condensation). When this extra term is included it can be shown that the magnitude of the angular momentum per unit mass in a stellar condensation is $a^2 \mid dV/dr + \alpha V/r \mid$. The coefficient α is > 0 and has a value that depends on the shape of the condensation. For a spherical condensation contracting under a radial gravitational force α is unity, whilst for a condensation of suitable elongation α can be made $<< 1$. The two terms dV/dr and $\alpha V/r$ are of the same sign in the main body, but are of opposite sign in the outer regions of a nebula where $dV/dr < 0$. The additional term modifies the remarks of 4.1 in that when $dV/dr < 0$ condensations can occur with small angular momentum provided $\alpha = -\frac{r}{v}\frac{dV}{dr}$; that is, provided the condensation is of an appropriate shape. For spherical condensations $\alpha = 1$, and condensation without restriction as to mass can occur if a value of r exists within the nebula such that $d(Vr)/dr = 0$; (it is possible that the outermost fringes may be reached before $d(Vr)/dr$ falls to zero).

‡ N. U. Mayall and L. H. Aller, *Ap. J.*, **95**, 5, 1942. § E. P. Hubble, *Ap. J.*, **97**, 112, 1943. A. B. Wyse and N. U. Mayall, *Ap. J.*, **95**, 24, 1942.

examination of this question (thereby confirming the earlier arguments of Slipher). This result is in sharp disagreement with the requirements of theories based on rotational instability which lead to the arms "advancing" in front of the nucleus. In addition the observations of Baade * and Seyfert † confirm the suggestion that the arms are only exceptional in that the bright stars are confined to these regions.

It would appear that the first step towards obtaining a theory of the spiral arms along the lines discussed above is to demonstrate that any one of the following possibilities will occur:—

(1) That massive stars will condense at a given time $\tau = 0$, say, along the diameter of the nebula given by the angles $\psi = \psi'$ and $\psi = \psi' + \pi$ and that no other massive stars will condense in the nebula.

(2) That the only stars becoming massive through accretion of interstellar material lie along the diameter given by the angles ψ', $\psi' + \pi$ at time $\tau = 0$.

(3) That although massive stars may occur throughout the nebula, only those stars that lie along the diameter given by the angles ψ', $\psi' + \pi$ at time $\tau = 0$ will undergo accretion (at times $\tau > 0$) of interstellar hydrogen at a sufficient rate to avoid exhausting their supply of hydrogen through thermo-nuclear reactions.

It is believed that of these possibilities (3) is the most probable, and a possible process in which (3) may be satisfied is given in 4.3 of Paper IV. The difficulty is not, however, to invent a process but to decide between the various possibilities that arise. This may be judged from the fact that according to the mass-luminosity relation a systematic difference in mass by a factor 3 between the stars in the arms and those outside the arms is sufficient to give a systematic difference in luminosity of about 5 magnitudes, which is sufficient to account for the enhanced brightness of the arms in the visible and ultraviolet parts of the spectrum. Thus a comparatively small systematic difference in mass at the time of condensation, or a small difference in the accretion rate would be sufficient. This means that second order processes may lead to the required effect, and it is always difficult in such problems to decide which, if any, is the dominant process. Although this situation is unsatisfactory there is one feature of the problem that appears to be independent of the particular process adopted. Thus the discussion of the shape and development of the spiral arms follows the same lines for any of the possibilities (1), (2) or (3) quite apart from what explanation is invoked to lead to one of these possibilities. Moreover, the discussion gives such a good account of the shapes of the spiral arms that it provides confirmation of the point of view adopted above. In these circumstances it seems worth describing this work without making any definite decision on the possibilities (1), (2) or (3).

Write $P(r)$ for the period of a star moving in a circular orbit at a distance r from the centre of the nebula. Suppose that the nebula is approximately axially symmetric and let the massive stars arising from one of the possibilities (1), (2) or (3) be moving in almost circular orbits. Then to a sufficient approximation the distribution of these stars at time τ is given by the equations

$$\left.\begin{aligned} \psi(r, \tau) &= \psi' + 2\pi\tau/P(r), \\ \psi(r, \tau) &= \psi' + \pi + 2\pi\tau/P(r). \end{aligned}\right\} \tag{27}$$

The equations (27) define the shape of the spiral arms at time τ. It is seen that the curvature of the arms depends on the form of the function $P(r)$. Thus if $dP/dr > 0$ the arms "trail", whereas if $dP/dr < 0$ the arms are "advanced". It is shown in Section 5 of Paper IV that dP/dr must be > 0 in stable nebulæ that are approximately axially symmetric. This result is confirmed by direct observation of the form of the rotational velocity function $V(r)$ in M31 and M33. It appears therefore that the arms must "trail" in all stable approximately axially symmetric nebulæ (axial symmetry means that the interstellar material is distributed with axial symmetry).

* W. Baade, *Ap. J.*, 100, 137, 1945. † C. K. Seyfert, *Ap. J.*, 91, 528, 1940.

In Fig. 1 the shape of the arms is given for a number of different cases. The amorphous inner nucleus is arbitrarily taken as having a radius of $r_{mb}/3$ where r_{mb} is the radius of the main body of the nebula. The cases $P(r)$ constant, $P(r) \propto r^{\frac{1}{2}}$ and $P(r) \propto r^{\frac{1}{3}}$ are considered in Fig. 1. These cases which correspond to $rF'/F = 1$, $\frac{1}{2}$ and 0 respectively are probably sufficient since, judging by the form of the function $V(r)$ in M31 and M33, it is reasonable to suppose that in general rF'/F will not be appreciably less than $\frac{1}{2}$ except in the outer regions of the nebulæ ($F(r)$ is the radial component of the

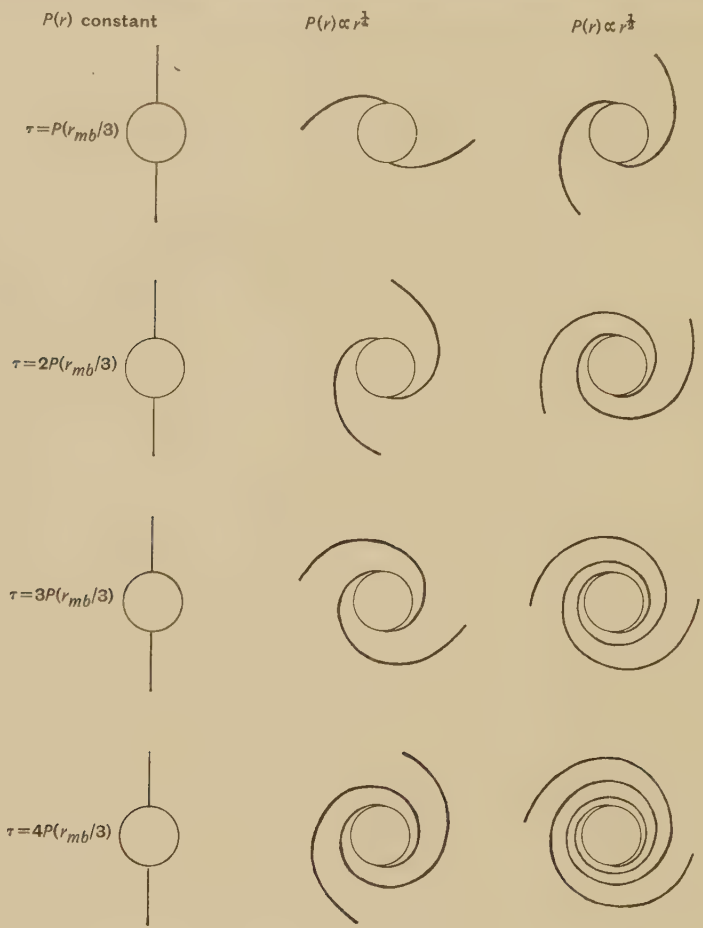


Fig. 1.

acceleration due to the gravitational field of the nebula). The examples given in Fig. 1 illustrate very well the various forms of spiral structure that are observed. In comparing Fig. 1 with the observed shapes of spiral, it will be remembered that the line of sight is taken in Fig. 1 as perpendicular to the plane of the nebula. It is seen that with increasing τ the spiral gradually winds itself up. For sufficiently large τ the spiral structure would be difficult to observe since the number of convolutions becomes very large.

The rotational velocity of M33 suggests a further application of equation (27). In the main body of M33 the rotational velocity $V(r)$ is approximately proportional to r. At a distance of about 1000 parsecs from the centre $dV/dr = 0$, and for still greater distances from the centre $V(r)$ decreases fairly rapidly with increasing r. The change from $V \propto r$

to $dV/dr = 0$ occurs in a small interval of r (that is, d^2V/dr^2 is very large at about 1000 parsecs from the centre). According to Wyse and Mayall * this change in the behaviour of $V(r)$ arises from a very rapid decrease in ρt at a radial distance of about 1000 parsecs. The amount of material lying outside this distance in M33 is probably insufficient to allow the following argument to be applied for this nebula. The present considerations apply to nebulæ with a similar rotational velocity curve to M33, but with a sufficient amount of material lying outside the $V \propto r$ region to form an appreciable number of stars. The case $P(r)$ constant of Fig. 1 will evidently occur in the $V \propto r$ region, whilst the discussion of 4.1 suggests that a bright ring of stars will be formed where $dV/dr = 0$. In the outer region where $V(r)$ decreases with increasing r the arms will trail. This leads to a distribution of bright stars of the form shown in Fig. 2. There are a number of nebulæ, NGC 986 for example, that show this general type of structure. As pointed out above, the formation of the ring and the outer trailing arms depends on there being sufficient material to form an appreciable number of stars. Cases intermediate between

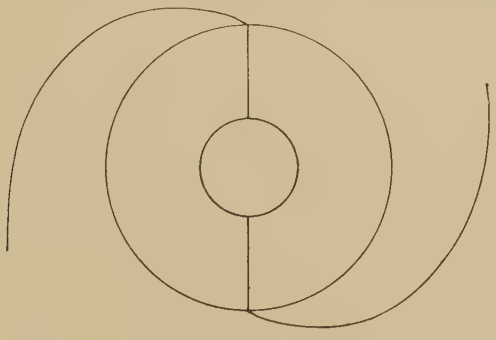


FIG. 2.

nebulæ satisfying this requirement and the sharp-edged nebulæ discussed by Wyse and Mayall are likely to arise in which the ring is broken or in which the outer trailing arms are either absent or very faint.

In addition to giving a satisfactory explanation of the above alternatives (1), (2) or (3), there is a further question that must be settled in a complete theory of the spiral arms. The bright luminous stars in the arms, with the exception of a group of red giant stars, are blue stars with surface temperatures that increase with increasing luminosity. These stars, which Baade † calls Type I, occupy the main series of the Hertzsprung-Russell diagram. On the other hand the stars outside the arms are probably of Baade's Type II, which also occur in globular clusters, elliptical nebulæ and as high velocity stars in the Galaxy. These stars have the remarkable property that their surface temperatures decrease with increasing luminosity and they occupy a peculiar position in the Hertzsprung-Russell diagram. The discussion of this question is evidently related to the theory of stellar structure. The problem is interesting and unusual because this is a case where work on stellar structure may give information of direct application to the problem of the distribution of the bright stars in the nebulæ.

It may be noted in conclusion that very bright stars are not likely to occur in the nuclei of the nebulæ, since the process discussed in Section 8 of Paper I requires the stars in the nuclei to have large velocities relative to the interstellar material in a direction perpendicular to the plane of the nebula. Thus accretion of interstellar material must be negligible in the nuclear regions, so that stars cannot become massive through accretion, Moreover, negligible accretion means that the lifetime of any stars of large mass that happen in exceptional circumstances to condense in the nuclear regions must be very

* *Loc. cit.*

† W. Baade, *Ap. J.*, 100, 137, 1945.

short. It is therefore to be expected that bright stars of large mass will be absent from the nuclei of the nebulæ.

Appendix

1. The problem of the condensation of stars under conditions in which new material is added to the condensation during the contraction process will now be discussed. In the first place we can dismiss the trivial case in which new material is pulled into the condensation by the action of the gravitational field of the condensation. Since this gravitational field cannot generate angular momentum about the centre of gravity of the condensation the discussion of this case evidently follows exactly the same lines as that given in Section 2. Thus if we define a as the radius of the sphere that contained before contraction all the material that finally joins the condensation, then we obtain the same equations as before.

The case that is of interest is when the condensation possesses a velocity, w say, relative to the surrounding interstellar material. It turns out to be possible to obtain a process that avoids the difficulties of Section 2, provided the velocity w is of order 10^4 cm. per sec. and is parallel to the plane of the nebula. It seems improbable that this process has widespread application, however, firstly, because there seems to be no reasonable explanation for the existence of such a velocity, and secondly, even if this condition were satisfied the process discussed below could only occur in a very small fraction of the total number of stellar condensations. Thus consider the first of these two objections.

The component parallel to the plane of the nebula of the gravitational field of the nebula cannot produce a relative velocity between the condensation and the surrounding interstellar material, since the gravitational acceleration acts equally on both the surrounding material and the condensation. Thus unless an external force is acting on the nebula the only force that could produce the required velocity is the gas pressure gradient. But in material in which stars are condensing the temperature must be less than 100° abs. (see Section 2), so that the *average* acceleration parallel to the plane of the nebula due to the gas pressure cannot exceed 10^{-12} c.g.s. units (see Section 6 of Paper I). This acceleration would have to act for a time of about 10^{16} sec. in order to produce a velocity of the required magnitude. Such a period is very much greater than the time of contraction of the condensation which is of order 10^{13} sec. (see below). It follows therefore that a velocity of required order of magnitude cannot occur unless one of the following requirements is satisfied:—

(1) The gas pressure has large local fluctuations giving gas pressure gradients that greatly exceed the average value discussed in Section 6 of Paper I.

(2) An external force acts on the nebula that gives a suitable differential acceleration between the condensation and the interstellar material.

Although it seems unlikely that either of these alternative requirements will occur, the possibility has to be kept in mind. For example, an external force of the type discussed in Paper IV may turn out to be of importance in this connection.

Before proceeding to the main discussion it may be noted that although there is difficulty in obtaining a velocity of order 10^4 cm. per sec. parallel to the plane of the nebula, it is easy to see that the condensation must in general acquire a velocity perpendicular to the plane of the nebula. Thus before the condensation forms, the interstellar material satisfies equation (1) of Paper I. As soon as a concentration of material occurs in a region with dimensions $<< t$ the gas pressure gradient is no longer sufficient to support the condensation against the component of the gravitational field of the nebula perpendicular to the plane of the nebula. Thus the condensation must move towards the central plane of the nebula unless it happens to be formed on the central plane itself. The velocity acquired by the condensation in this process will probably be of the same order as the velocity of sound in the interstellar material, which for $T_H \simeq 100°$ abs. is of

order 10^5 cm. per sec. It is shown below that the existence of such a velocity is sufficient to invalidate the process described in the following investigation.

2. We now consider the angular momentum of the material in the condensation. In the following discussion the condensation will be regarded as having a characteristic dimension a (the shape not necessarily being spherical). In order to avoid the difficulties of Section 2 we require the motion of the condensation relative to the interstellar material to cancel the angular momentum arising from the differential rotation of the interstellar material. The angular momentum per unit mass arising from the differential rotation has a magnitude (see Section 2) of order $a^2 \mid dV/dr \mid$ about an axis through the centre of gravity of the condensation taken perpendicular to the plane of the nebula. It appears therefore that the velocity w between the condensation and the interstellar material must give

(a) A component of angular momentum about an axis through the centre of gravity of the condensation perpendicular to the plane of the nebula, that has a total magnitude of order $Ma^2 \mid dV/dr \mid$.

(b) The angular momentum about axes parallel to the plane of the nebula must be small compared with $Ma^2 \mid dV/dr \mid$.

The second of these requirements is necessary since otherwise the motion between the condensation and the interstellar material would itself introduce difficulties of the same character as were discussed in Section 2. The requirements (a) and (b) are necessary, but are evidently not sufficient since we also require the angular momentum (a) to be of opposite sign to the angular momentum arising from the differential rotation of the interstellar material. These requirements will now be discussed.

Let ϵ, λ, η be a system of Cartesian co-ordinates with origin at the centre of gravity of the condensation, and let the ϵ-axis be taken parallel to the direction of the relative motion between the condensation and the surrounding interstellar material. Then the rate at which angular momentum is added to the condensation is

$$\left.\begin{array}{ll} -\rho w^2 \int_S \lambda d\lambda d\eta & \text{about the } \eta\text{-axis,} \\ \rho w^2 \int_S \eta d\lambda d\eta & \text{about the } \lambda\text{-axis,} \end{array}\right\} \tag{28}$$

where S is the area given by projecting the condensation on the plane $\epsilon = 0$. There is evidently no angular momentum about the ϵ-axis. It follows therefore that the requirement (a) cannot be fulfilled if the ϵ-axis is perpendicular to the plane of the nebula.

Consider the general case in which the ϵ-axis is inclined at an angle θ to the plane of the nebula, and let the λ-axis be chosen to lie in the plane of the nebula. Then the three components of angular momentum have the following magnitudes:—

(i) $\mid \rho w^2 \cos\theta \int_S \lambda d\lambda d\eta \mid$

perpendicular to the plane of the nebula,

(ii) $\mid \rho w^2 \int_S \eta d\lambda d\eta \mid$

about the λ-axis,

(iii) $\mid \rho w^2 \sin\theta \int_S \lambda d\lambda d\eta \mid$

about an axis perpendicular to both (i) and (ii). If τ is the time required for appreciable contraction of the condensation to occur, then the total angular momentum about these axes is given by multiplying (i), (ii) and (iii) by τ. Thus in order to satisfy the requirements (a) and (b) we must have

$$\begin{aligned} \mid \tau\ \rho w^2 \cos\theta \int_S \lambda d\lambda d\eta \mid &\simeq Ma^2 \mid dV/dr \mid, \\ \mid \tau \rho w^2 \int_S \eta d\lambda d\eta \mid &\simeq 0, \\ \mid \tau\ \rho w^2 \sin\theta \int_S \lambda d\lambda d\eta \mid &\simeq 0. \end{aligned}$$

It follows immediately, in order to satisfy these conditions, that

$$\int_S \eta d\lambda d\eta \simeq 0, \tag{29}$$

$$\theta \simeq 0, \tag{30}$$

$$\tau \mid \rho w^2 \int_S \lambda d\lambda d\eta \mid \simeq Ma^2 \mid dV/dr \mid. \tag{31}$$

The condition (30) requires the relative velocity between the condensation and the interstellar material to be effectively parallel to the plane of the nebula, which leads immediately to the following difficulties:—

(*a*) As shown above, there is no clear explanation for the origin of such a velocity.

(*b*) As shown above, it is to be expected that a condensation will in general have a velocity relative to the interstellar material in a direction perpendicular to the plane of the nebula.

Thus even if there were also a component of velocity parallel to the plane of the nebula it would still not be possible to have $\theta \simeq 0$ unless the component parallel to the plane of the nebula were large compared with the component perpendicular to the plane of the nebula. This requirement cannot be satisfied unless the condensation happens to be formed very close to the central plane of the nebula, in which case the velocity perpendicular to the plane of the nebula will be small (otherwise this component of velocity will be of the order of the velocity of sound in the interstellar material).

Consider finally the condition (31), which can be used to obtain a numerical estimate for w. An order of magnitude estimate for τ may be obtained from $1/\{2\sqrt{G\rho}\}$, which is the time required for appreciable contraction to occur in a spherical condensation containing material of density ρ when gas pressure forces are neglected. The effect of gas pressure forces will be to increase the value of τ, but so far as an order of magnitude is concerned it is reasonable to suppose that τ can be estimated from $1/\{2\sqrt{G\rho}\}$. If we use the estimate $\rho = 1{\cdot}1 \,.\, 10^{-19}$ gr. per cm.^{3} obtained above in Section 2 the value of $1/\{2\sqrt{G\rho}\}$ is $5{\cdot}8 \,.\, 10^{12}$ secs. It follows that $\tau \simeq 10^{13}$ sec. gives a reasonable estimate for the time of contraction (this slight increase in τ above $5{\cdot}8 \,.\, 10^{12}$ sec. makes some allowance for the gas pressure forces).

For the purpose of considering (31) it is convenient to write

$$\mid \int_S \lambda d\lambda d\eta \mid = \delta \pi a^3. \tag{32}$$

The value of δ may lie anywhere between zero and a value of order unity according to the shape of the condensation. Thus when the condensation is symmetrical about the plane $\lambda = 0$ the value of δ is zero, while δ is of order unity if there is a very marked asymmetry about the $\lambda = 0$ plane. If (32) is used in (31) together with

$$M \simeq 4\pi a^3 \rho / 3$$
$$\mid dV/dr \mid \simeq 2.10^{-15} \text{ sec.}^{-1}$$

we obtain

$$w \simeq 1{\cdot}6 \,.\, 10^{-14} \delta^{-\frac{1}{2}} a. \tag{33}$$

Now equation (9) of Section 2 shows that a is of order $1{\cdot}7 \,.\, 10^{17}$ cm., so that (33) gives

$$w \simeq 3.10^3 \delta^{-\frac{1}{2}}, \tag{34}$$

which is a condition between w and δ. That is, there is a condition between w and the degree of asymmetry of the condensation about the $\lambda = 0$ plane. The value of w must evidently in general be of order 10^4 cm. per sec.

It is seen from the above discussion that the relative velocity between the condensation and the interstellar material must not only be parallel to the plane of the nebula, but the

magnitude of the velocity must be related to the degree of asymmetry of the condensation about the $\lambda = 0$ plane. Thus since only a small proportion of condensations will possess this relation between w and δ it follows that the present process, if it applies at all, cannot affect more than a small fraction of the stars.

[*Note added in Proof.*—For convenience in printing it has been decided to defer Papers III and IV to a future issue of *Monthly Notices*. The end of Paper II forms a natural break in the discussion for at this point there is a change from general argument to a more detailed investigation of special problems.]

St. John's College,
Cambridge:
1945 *October* 30.